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ABSTRACT

Twenty third-year students from two rural teachers' colleges in Jamaica carried out a coordinated set of individual investigations into children's learning of geometrical concepts during their assignment to schools as intern teachers. All the studies, which covered classes from grade 2 to grade 10, included some initial testing of children's knowledge of a selected topic, the teaching of an experimental unit on that topic, and a post-test to measure what had been learnt; several investigators also interviewed other teachers on their attitudes toward geometry teaching. This report includes abstracts of each teaching study and summarizes the results in terms of children's present knowledge of geometry; their ability and motivation to learn geometry; some linguistic, logical and spatial difficulties they experience; problems which the teacher has to face in organising concrete activities; and some possibilities for integrating geometry with arithmetic and other school subjects. A final section summarizes the findings of the teacher interviews. The general conclusion from the 20 studies is that younger children know very little geometry and older children find the learning of geometry difficult largely because very little geometry is taught in primary schools. (Author)

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FINAL REPORT

of the

COOPERATIVE GEOMETRY RESEARCH PROJECT

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ABSTRACT

Twenty third-year students from two rural teachers' colleges in Jamaica carried out a coordinated set of individual investigations into children's learning of geometrical concepts during their assignment to schools as intern teachers. All the studies, which covered classes from Grade 2 to Grade 10, included some initial testing of children's knowledge of a selected topic, the teaching of an experimental unit on that topic, and a post-test to measure what had been learnt; several investigators also interviewed other teachers on their attitudes toward geometry teaching. This report includes abstracts of each teaching study and summarises the results in terms of children's present knowledge of geometry; their ability and motivation to learn geometry; some linguistic, logical and spatial difficulties they experience; problems which the teacher has to face in organising concrete activities; and some possibilities for integrating geometry with arithmetic and other school subjects. A final section summarises the findings of the teacher interviews. The general conclusion from the twenty studies is that younger children know very little geometry and older children find the learning of geometry difficult largely because very little geometry is taught in primary schools; but it is possible to teach geometry in a concrete, exploratory mode which can lead, despite initial difficulties, to enjoyable creative activities, gains in social cooperation, and considerable learning.

I. ACTIVITIES OF THE PROJECT

Evidence has been accumulating for some time that the average Jamaican schoolchild finds geometry the most difficult area in the already difficult subject of mathematics. Various reasons have been proposed for this state of affairs, ranging from a suspected lack of geometry teaching (especially in primary schools) to an inherent handicap in visualization. The Co-operative Geometry Research Project was established in order to obtain more information on the situation, and especially to answer these two questions:

1. What geometry do Jamaican children of different ages know?

2. How much geometry can they learn?

It was also hoped to gain some insight into the reasons for children's difficulties and to identify the most successful teaching strategies.

The Project consists of 20 coordinated studies carried out by third-year students at Bethlehem and Church Teachers' Colleges in January-February 1981. The students were attached to various schools as interns throughout their third year, and were required as part of their course to complete an "individual study" during that time. The topics of these studies were decided during group seminars, but the studies remained the responsibility of individuals, guided in the usual way by their college lecturers.

Organization of the Project

The Project Director was Dr. M. C. Mitchelmore, Senior Research Fellow in the Teaching of Mathematics at the U. W. I. School of Education, Mona. In May 1980, he proposed the Project to the Heads of the Mathematics Departments at Bethlehem and Church Teachers' Colleges and asked them to ascertain reaction among their students, especially those who had shown special interest in geometry. The response was enthusiastic; many more students could have been included had there been more funds available. It was finally decided to limit the numbers to 20.

During the course of the Project, three seminars were held, all at the Hotel Astra, Mandeville. At the first seminar held on 26 September 1980, the Project Director first outlined the problem.

After presenting some figures extracted from item analyses of public examinations at secondary level and reviewing several teaching projects carried out by Cert. Ed. and Dip. Ed. students at Mona, he outlined the van Hiele theory of conceptual development in geometry and suggested that the main reason for secondary school students' poor performance was that they were being required to think analytically at a time when they were still processing geometrically ideas globally. Several teaching activities were then described which could help children develop their global ideas and could also be used to introduce more analytical notions. The paper presented at this opening session is included as an appendix to this report.

The Director next outlined possible research methods which the participants could use. He suggested that each investigator should, after choosing a suitable topic in geometry, pretest a group of children to ascertain their present knowledge of the topic; then design an appropriate teaching unit, depending on the findings; and finally administer a post-test to determine what children had learnt. Questions of data analysis, the testing of comparison groups, and some ideas for collecting supplementary data, were then briefly discussed.

After an informal session during which participants consulted resource materials and discussed ideas with their college lecturers, the topics for the 20 studies were decided. Some topics were to be investigated in a similar manner at different grade levels, in order to compare children's reactions. In other cases, a topic was taught in an introductory manner at one grade level and taken further by another investigator at a higher grade level. At this stage, some participants changed their topics in order to achieve a better spread of topics across grade levels.

The second seminar was held on 27-28 November 1980 to discuss participants' preliminary results. Investigators were each given 15 minutes to describe their pretest findings, outline their proposed teaching unit, and respond to comments and suggestions. There were also several hours set aside for reading and consultations with the Project Director and college lecturers. The third seminar, held on 26-27 February 1981, followed a similar pattern, except that participants described the outcomes of their teaching units. The final session at this seminar was devoted to summarising the results of the 20 studies under various headings and to evaluating the Project.

The completed individual studies were handed in to the office of the Joint Board of Teacher Education in May 1981; participants made copies of their studies to allow the present report to be written after the originals had been returned to their colleges. The list of participants and the titles of their studies are given below. All the studies were carried out in schools in Westmoreland, St. Elizabeth, Manchester and Clarendon; individual school names appearing in study titles have been deleted to preserve anonymity.

Bethlehem Teacher's College

DOUGLAS, Cheryl M. The teaching of some quadrilaterals to a Grade Six class at *** Primary School in Westmoreland.

JAMES, Fadia. A study of the teaching of some quadrilaterals to a Grade 4 class at *** Primary School in Westmoreland.

LAWRENCE, Lesma. A study on the teaching of mirror symmetry to a Grade Five class at *** Primary School in Westmoreland.

McLEAN, Halcon. Teaching of angles to a Grade Seven class of a new secondary school in Western Jamaica.

MORRISON, Argelyn. A study on how much Grade Six children know about area and how much they can learn on the subject.

PUSEY, Marva. The teaching of rotational symmetry to a group of Grade Nine second stream students.

SAMUELS, Shervette E. What pupils of Grade 6^4 at *** All-Age School know and can learn about mirror symmetry.

SMITH, Vinette. The teaching of some plane shapes to a group of children in Grade 5^2 at the *** Primary School.

WEBSTER, Monica W. Copying and making patterns at the Grade Three level of *** Primary School.

WEDDERBURN, Paulette A. A study of the extent to which geometry is taught in primary schools with special reference to the teaching of squares, rectangles and perpendiculars at the *** All-Age School.

WEST, Verona. The teaching of geometry with special reference to the teaching of circle to pupils of Grade 8 at *** Secondary School.

Church Teachers' College

ANDERSON, Lennox. Tessellation - a practical approach to the teaching of geometry in the primary school.

CUNNINGHAM, Phillipene. Mathematics teaching and pupil performance among a selected group of Grade Four pupils in a primary school.

GOLDING, Elsa V. An investigation into the geometrical knowledge of Jamaican primary school children in a Grade Two class.

LARGIE, Worrell. The teaching of linear measurement to a particular Grade 10 class in a rural secondary school in Jamaica.

LAWRENCE, Vivienne M. Performance in geometry among a selected group of Grade Six pupils in a Jamaican All-Age School.

LISLE, Micella H. How much do Grade Three children know about symmetry? How much can Grade Three children learn about symmetry?

MARTIN, Rose M. Geometric performance with special emphasis on solid geometry among a group of Grade Three students in a rural primary school.

McLEOD, Millard C. Making and drawing common solids.

NELSON, Marjorie E. Geometric performance among a selected group of Grade Four pupils in a Jamaican primary school, using symmetry as a base.

This Report

The major aim of this report is to summarise the findings of the 20 individual studies. Chapters 2 and 3 present the results of the teaching studies, Chapter 2 giving the details of each study and Chapter 3 drawing out the common findings and examining their implications. Chapter 4 summarises the results of interviews with primary school teachers, an unstructured exercise undertaken in 12 of the 15 studies done in Grades 2-6. An Appendix reprints a resource paper supplied to participants and a second gives a classified listing of the literature made available to participants at the seminars.

In summarising the individual studies, the author was aware of several factors threatening the validity of the whole exercise.

Firstly, the study reports were of a variable quality, often failing to give what would normally be considered as essential details of their research (e.g. the test items, mean scores, or numbers of teachers interviewed). Also,none of the studies used carefully constructed and validated tests, and none included any measures of retention. These shortcomings were inevitable in the circumstances, since it was felt that it would be unfair to other third year students to give the participants in this study rigid direction or special tuition in test development or research reporting.

Secondly, the investigators were all biased towards geometry teaching, as is the Project Director; there is little doubt that, as a result, favourable observations tended to be reported and abstracted more often than unfavourable occurrences.

Thirdly, all the teaching was done in untypical conditions: the teachers were fresh from college and they had an abbreviated timetable, small classes and temporary status, were in regular contact with their supervisors, and knew that they were to be assessed on their work.

To defend against the effect of these factors, we shall not report results obtained in individual studies if their validity is in any doubt and we shall concentrate in the summary on findings which appear in several studies. We recognise the possibility of bias and do not claim that similar results would be easily obtained in every classroom. Rather, we present the results as hypotheses to be tested in more rigorous studies and

as challenges to the school system to create conditions under which children can learn geometry.

Evaluation of the Project

The student participants were most favourable in their comments on the Project. They valued the presence at seminars of resource materials and persons not available to them at their colleges in exposing them to a wider range of ideas (particularly in enabling them to understand the need to progress from concrete activities to abstract ideas) and in providing suggestions for teaching activities. About 50% referred specifically to the value of the group discussions in stimulating thought and in helping them learn to give and receive positive criticism, and a similar percentage indicated that participation had developed their interest in geometry, changed their perception of the subject, or helped them recognise its importance in general education. Most of the participants also felt that their studies had been valuable to them personally in making them aware of the potential for geometry teaching, of children's ability to learn geometry, and of teachers' generally negative attitude to the subject.

Very few negative comments were received. About a third of the participants would have preferred more time for their study, but this was not possible within the constraints of the third-year programme. Several wished there had been one further seminar to discuss the final results, and more visits to them during the course of their study; these were not possible within the constraints of the Project budget. A few participants suggested changing the venues for the three seminars, to provide variety.

College lecturers felt that the Project had greatly helped third-year students by providing them with a structure within which to carry out their studies; having decided their topic early in the first term, participants were far better placed than their colleagues not in the Project, who often spent the whole of the first term looking around for a topic with no clear idea of the sort of problem they could be attacking.

From the point of view of the generation of knowledge, the reader may judge the value of the Project from the material presented in the following pages. Without the Project, each individual study would have been viewed purely as a learning exercise, and the results, which by the nature of the exercise would have had dubious validity, would never have seen the light of day.

By coordinating the studies so that similar topics were investigated with different groups of children by different researchers, the shortcomings of the individual efforts have to a large extent been overcome.

It is the writer's (subjective and biased) opinion that the Project was extremely good value for the J$6,000 expended. The 20 participants may, if they are not overwhelmed by the conditions in their new schools or driven out of the profession for financial reasons, be the seed from which a completely new attitude to geometry will develop. The scientific results may generate methods and materials which could be of great help in furthering this development. It is suggested that the model of cooperative research followed in this Project could provide a most beneficial and profitable means of encouraging and coordinating the many pieces of individual research which are carried out in this country in different institutions and at different levels of sophistication, and it is recommended that opportunities for further cooperative research projects in other areas of pressing need should be actively sought.

Acknowledgements

This Project would not have been possible without the willing participation of several college mathematics lecturers, especially Mr. L. Wright of Bethlehem Teachers' College and Mr. H. Thompson of Church Teachers' College, who attended all the seminars and often acted as links between the Director and the Project participants. Thanks also go to lecturers Mrs. Spence of Bethlehem and Mrs. Baker and Mrs. C. Holzwarth of Church. A further important role was played by the supervisors of third-year students at the two colleges, Mrs. R. Anglin, Mrs. Falconer and Mr. G. Daley of Bethlehem and Mr. H. Brown of Church, and the Project was greatly assisted by principals' willing agreement to allow the participants to teach the topics selected for their studies. We thank also Mr. I. Isaacs, Mr. R. Sigsworth and the Documentation Centre of the School of Education, Mona, for lending resource literature. The management and staff of the Hotel Astra were always pleasant and helpful. Finally, we must not forget to thank all the teachers, parents and students who assisted the investigators in various ways.

2. ABSTRACTS OF TEACHING STUDIES

Each participants' study included a test of an experimental teaching unit, preceded by a pretest to assess initial knowledge and guide unit planning and followed by a posttest to assess the effectiveness of the unit. In most cases, the pre- and posttest were identical (exceptions are noted below); some investigators also tested comparison groups who had not been taught the unit. This chapter reports the various teaching studies, grouped in seven sections: Basic shapes, Fitting shapes together, Parallels and perpendiculars, Mirror symmetry, Angles and rotational symmetry, Solid shapes, and Measurement.

Basic Shapes

Two investigators taught children to recognise the circle, square, rectangle and triangle: Golding in Grade 2 and Cunningham in Grade 4.

Golding Pupils had met the basic shapes in Grade 1; in the pretest, the circle was correctly identified by 100% of the 26 pupils, the square by 54%, the rectangle by 42% and the triangle by 73%; children most often mixed up triangles with rectangles and rectangles with squares. The average score on four items asking pupils to name shapes with given properties (e.g. three sides) was 45%. Performance in a comparison class from a higher stream was only slightly better.

The teaching unit consisted of 10 lessons spread over 4 weeks. After an initial lesson on sorting shapes, children examined in turn squares, rectangles and triangles and described the properties of each shape; named everyday objects with these shapes; completed work sheets on the properties of each shape and the difference between the three shapes; and learned the difference between straight and curved sides. The teacher gave out cardboard models of each shape in which each side had a different colour; the pupils then traced around the shape and did turning exercises to discover equal sides and "corners". It was seen that a triangle has the same number of equal sides as equal corners, but the terms scalene, isosceles and equilateral were not used. In art lessons, pupils made pictures of familiar objects using the four basic shapes.

There were some interesting reactions to the classification exercises. In the first lesson, fat and thin rectangles were put into different groups, and right-angled triangles were often separated from other triangles; children said they had grouped shapes which looked the same. However, having accepted that all the triangles should be in the same group because they all have

three sides, there was then some reluctance to accept that different triangles could have different numbers of equal sides - some pupils even changed their answers to agree with those of the "brighter" children. Otherwise, children seemed to have little difficulty with this unit.

On the post test, done two weeks after the unit was completed, each basic shape was identified by at least 92% of the class, and the average score on the other four items (identical to the pretest) was 98%. Scores in the comparison group were little different from the pretest. Clearly the experimental group had learned what Golding set out to teach; furthermore, she reports, pupils had become more eager to do mathematics each day, there had been an improvement in participation and cooperation in group work, and many of the below-average children had gained in self-confidence.

Cunningham This unit also extended over four weeks and covered the classification, naming and construction of the basic shapes; the identification of these shapes in the environment; their properties in terms of the number and size of their sides and corners; and the names and properties of various types of triangle. The 20 children in her Grade 4 class worked mainly in small groups, using cardboard cut-outs, cord, sticks and straws, squared paper, the geoboard, and models of some common solid shapes. Pupils were expected to help by bringing in discarded materials for use in class.

The mean score on a unit test increased from 35% before the unit was taught to 77% after; the 9 boys and 11 girls made approximately equal gains. Pupils who were rated above-average in bringing material to school and participating in class activities scored an average of 19% higher than those rated below-average. Similarly, those judged to be of high socioeconomic status scored an average of 17% higher than those of low S.E.S.

Fitting Shapes Together

Three studies treated the way shapes (especially the square, rectangle and triangle) can fit together to cover the plane, leading up to the measurement of area; Webster investigated pattern making in Grade 3, Anderson taught tessellations in Grade 6 and Morrison dealt with the measurement of area in Grade 6. These studies also touched on various properties of the basic shapes.

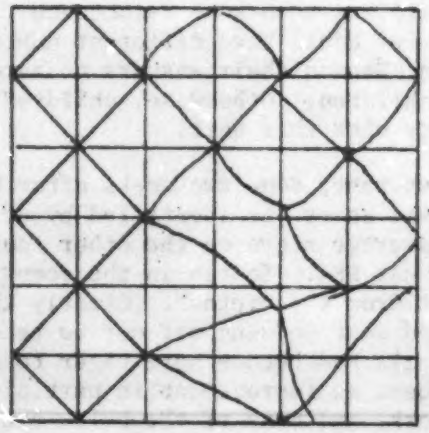

Figure 1 A Grade 3 pupil's attempt to continue the pattern given in the two left-hand columns. From Webster's pretest.

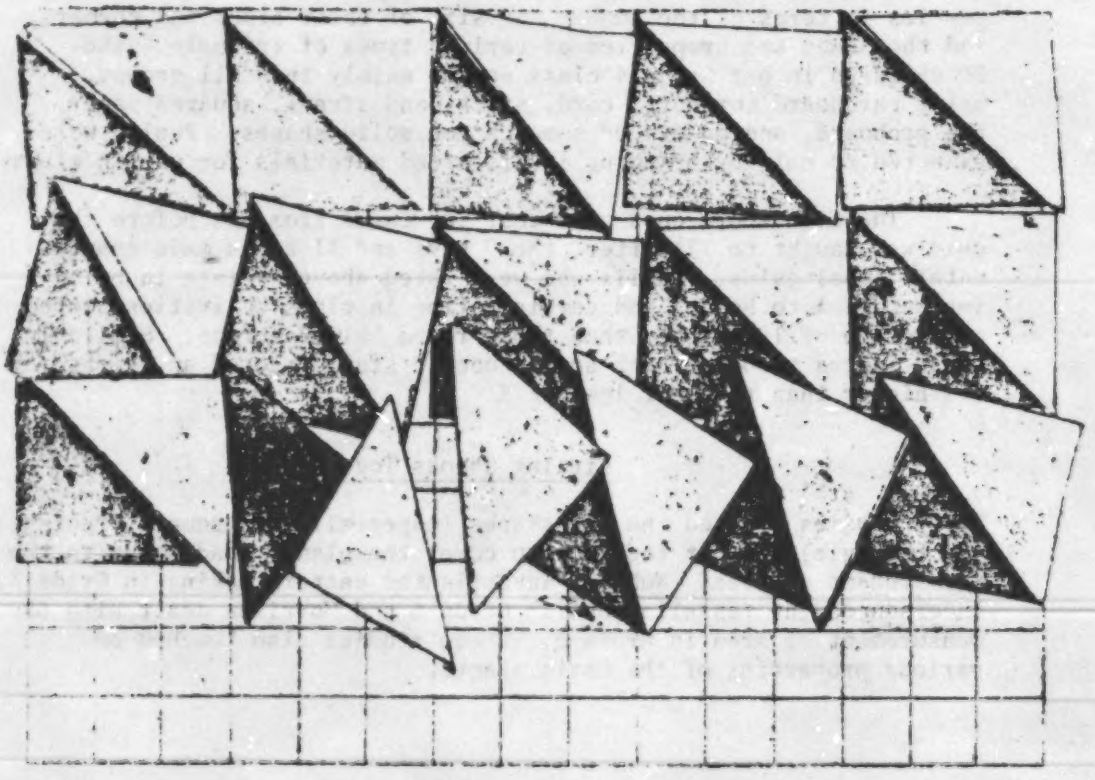

Figure 2 A Grade 6 pupil's attempt to stick right-angled triangles on to a square grid to form a tesselation. From the second week of Anderson's teaching unit.

Webster In the pretest, a circle was correctly named by all the 25 pupils in her Grade 3 class, and a square, a rectangle and a triangle by 62-64%. However, knowledge of patterns was very weak - only one pupil could point to a pattern in the classroom or name one outside, pupils made rather poor efforts at continuing given patterns, and only three children could create their own pattern. Figure 1 illustrates pupils' difficulties in continuing patterns.

The teaching unit consisted of 23 lessons over four weeks - ten from Mathematics, twelve from Art and Craft and one from Language Arts. The art lessons consisted of identifying patterns in the classroom and during a nature walk, and then making patterns using junk material, string pulling, paper folding, drawing, blobbing, paint blowing, block printing, tie-dying, and weaving. The emphasis here was on the regular repetition of a basic shape to create a pleasing pattern. (Naturally, many of the patterns were rectangular.) The mathematics lessons dealt with different types of triangles, the difference between a square and a rectangle, finding perimeters, making tessellations of squares and rectangles and finding rectangular areas by counting squares. Pupils also wrote a composition on "Patterns".

After an anxious start, children became very eager and worked carefully trying to create "better" patterns than their friends. They brought materials and patterns from home and talked freely about their discoveries; no stigma seemed to attach to using junk materials. It was noted, however, that children preferred to copy patterns freehand rather than use tracing paper. Also, some children had difficulty dividing rectangles into squares in order to find their areas.

On the posttest, all the children named all four basic shapes correctly and many identified patterns in their environment. There was still difficulty in continuing given patterns, but "after much effort" about 80% produced adequate completions. Pupils showed much enthusiasm and creativity in constructing their own patterns. The comparison with the pretest shows that students had made enormous progress. The investigator also reports that pupils seemed to have become more alert, observant, creative and willing to "experiment and research".

Anderson The 18 pupils in this Grade 6 class were given both a written and an oral pretest. On the written test, children were asked to draw various figures on squared paper, to say whether given shapes could form a tessellation, to draw tessellations using a given rectangle and square, and to find the area of four rectilinear shapes drawn on a square grid. The mean score was 8%. On the oral test, pupils were shown cards with diagrams of several shapes and asked to point to all the examples of various named shapes. Performance here was surprisingly poor, the best-known shapes being

the right-angled triangle (55% correct), the square (33%) and the rectangle (33%). There was frequent confusion between the square and the rectangle; 33% called a rectangle a square. Similar frequencies were found when pupils were asked to name the shape forming a given tessellation.

Teaching consisted of 15 one-hour lessons spread over four weeks, and fell into three sections. In the first, designed to reinforce concepts of basic shapes (various triangles, square, rectangle , pentagon and hexagon), children outlined shapes on squared paper, drew shapes to given dimensions, and made shapes on geoboards. The second section dealt with tessellations of these shapes, firstly by sticking card squares or triangles onto squared paper and then by drawing given tessellations on squared paper and creating original ones; children were also encouraged to produce a colour pattern using card shapes of different colours or by crayoning their tessellations. In the final lesson, children learned how to find the area of a rectangle by dividing it into unit squares and counting the squares.

Children's work collected during the teaching period showed evidence of considerable individual differences and much learning. Elementary errors included pencil lines not following the grid lines or made up of two distinct segments; vertices not on a grid vertex; sides of the wrong length; square cards not quite fitting on the grid lines, so that adjacent squares did not match; card shapes rotated so that they cannot possibly fit together (see Figure 2 for an illustration); and drawn shapes fitting together but changing shape from one side of the paper to the other. These errors gradually disappeared as pupils did further work on each topic. The major remaining difficulty appeared to be in drawing isosceles triangles on squared paper; there was apparently no discussion of symmetry in this unit.

The posttests were identical to the pretests. On the written test, the mean score was 59%, an increase of 51% over the pretest. On the oral test, only one pupil (who had not been in regular attendance) could not identify all the shapes on the cards; the shapes in triangular, rectangular and square tessellations were identified by 83%, 89% and 100% of the pupils respectively. Pupils had obviously learned a great deal not only about the basic geometrical shapes but also about shape in general.

Morrison This study, done with a group of 8 pupils selected from a Grade 6 class, was intended to deal with area but was largely concerned with teaching children to appreciate how certain shapes can fit together.

On the written pretest, half the children gave the correct values for the areas of various squares and rectangles, but none could state how many unit squares would fit into each shape. A group of 10 Grade 7 children were also tested; they all found the correct areas but again none could state the number of unit squares in each shape. Morrison concluded that some pupils had learnt the

"length x breadth" formula but none had any concept of area measurement.

The 4-week teaching unit started with exercises on covering regions with congruent shapes in a rectangular pattern; books were covered with bottle caps, desk-tops with books, and the classroom with desks. The rectangular pattern was highlighted by looking at squared paper and by pulled-thread activities with crocus bag material. It took some time for children to appreciate that the covering pattern is formed by intersecting, regularly-spaced perpendicular lines; and they counted squares one by one, not using multiplication without instruction. The general idea of area measurement was reinforced in art and craft lessons; for example, children made irregular shapes representing people, houses etc. on squared paper and counted the number of squares covered; or they covered a sheet of squared paper in a tessellation, coloured it in a pattern and counted the number of unit shapes of each colour. Areas of triangle and circles were also found by counting squares. Eventually, children were able to estimate the areas of objects in the classroom.

Pupils nevertheless had difficulty in proceeding to the semi-concrete case; they did not find it easy to draw the squares in a rectangle of a given size, and they still counted the squares one by one. Towards the end of the teaching period, a few pupils noticed that there were the same number of squares in each row and in each column when a rectangle was divided into squares.

Performance on the posttest was much improved, with about two-thirds of the pupils showing understanding of the items testing their concepts of area measurement. The most frequent error was the use of linear units instead of area units.

Parallels and perpendiculars

Four investigators taught rather similar units on parallel and perpendicular lines and the corresponding properties of squares and rectangles: James and Wedderburn in Grade 4, Smith in Grade 5 and Douglas in Grade 6. This topic follows logically after the two topics previously discussed, which were largely concerned with developing childrens' ideas of what different shapes look like; it begins an analysis of shapes in terms of geometrical relations. However, given the present state of geometry teaching in Jamaican schools, it is likely that pupils will have had only a little exposure to a topic such as "Basic shapes" before the present units were taught.

James A written pretest was given to all 36 pupils in her Grade 4 class. The results suggested mostly guesswork on drawing and labelling points, lines and line segments and identifying parallel and perpendicular lines; only 31% correctly selected the square and 22% the rectangle from given diagrams of a square, rectangle and triangle. The mean score was 18%.

A unit of seven lessons spread over four weeks was then taught to a random half of the class. Pupils identified squares and rectangles in the classroom and cut out examples from magazines; drew points, lines and line segments and learned how to label them; identified parallel and perpendicular lines in the environment and drew and made them with cardboard strips; and examined parallels and perpendiculars in squares and rectangles. The rhombus was discussed after a square was accidentally distorted. There was one geoboard used for demonstration; otherwise, children mainly used scrap materials.

The 18 children who were taught this unit increased their mean score on the posttest to 88%; the mean score of the other 18 children was unchanged.

Wedderburn The 27 pupils in this Grade 4 class were given a short oral pretest. Only 3 or 4 could name an object resembling the shape of a given card square or rectangle, tell what is a square or a rectangle, or identify from a display showing six shapes the ones which resembled the given card square or rectangle. In the last-named item, many pupils selected only the larger figures in the display, and some actually said that the smaller figure of a square was not a representation of a square.

The teaching unit consisted of 18 lessons over 6 weeks, and covered perpendiculars but not parallels. Children used observation, drawing and various concrete materials to learn about points, lines, line-segments, rays and angles (as corners). They made a paper set-square and used it to test for perpendiculars in the classroom. They collected examples of squares and rectangles, made patterns with them by sticking them onto paper and by drawing them, and listed their properties. Games, geoboards and a flannel graph were used at various points, and examples of pupils' work were displayed in the mathematics or art corner of the classroom. The investigator reported considerable enthusiasm for the work and development of childrens' powers of observation.

In the posttest, at least 80% of the pupils gave correct answers to each item.

Smith The mean score of the 17 pupils in her Grade 5 class on an 18-item pretest covering the topics in the teaching unit described below was 22%. All the pupils identified a square but none could define it; there was obvicusly much guessing on most of the other items.

The unit, taught in seven 50 minute lessons over four weeks, covered points, lines, line segments and rays; parallel and perpendicular lines; squares and rectangles; and the parallelogram and rhombus. Children found examples of each shape or relation both inside and outside the classroom; used card strips, a geoboard and drawing for constructions, and string to compare lengths; and made patterns and pictures in art lessons using the various shapes discussed in mathematics.

In the posttest, almost all pupils answered almost all items correctly; the mean score was 92%.

Douglas The pretest, taken by 23 children in this Grade 6 class, covered labelling of lines and line segments, the identification of parallels and perpendiculars, and the names of basic shapes. The mean score was 43%, suggesting that pupils had met many of the ideas before.

The teaching unit, which was taught in six 60 minute lessons over six weeks, covered the same topics as Smith's unit just described, but made greater use of class discussion and drawing activities and less use of other concrete materials. The mean score on the posttest, for which 6 pupils were absent, was 54%. The investigator attributed the modest improvement to poor attendance, noting that those who attended school regularly made more improvement than those with very poor attendance.

Mirror Symmetry

Four studies investigated the teaching of mirror symmetry: Lisle in Grade 3, Nelson in Grade 4, L. Lawrence in Grade 5 and Samuels in Grade 6. All units taught an intuitive recognition of symmetrical figures, and two included elements of analysis and application to properties of the basic shapes.

Lisle Written and oral pretests were given to the 17 children in this Grade 3 class. The written test asked children to identify figures which could be "divided equally", to draw the dividing line, and to complete symmetrical figures. All children scored less than 30%, for a mean of 12%. On the oral test, children were to select the symmetrical figures from given shapes; 16 of the 17 children could only find one out of four symmetrical shapes.

The teaching unit lasted for four weeks and covered recognition of symmetrical figures, drawing mirror lines, completing symmetrical figures, and the symmetry properties of squares, rectangles and triangles. Children made a field trip to find symmetrical figures and carried out many practical activities involving paper folding and tearing and paint blobbing.

Once the idea of symmetry had been grasped, children were meticulous in applying it; they would not accept that a leaf was symmetrical when its outline was symmetrical but the veins were not. Drawing created some problems: children had to be taught how to draw a straight line with a ruler. They found it difficult to complete symmetrical figures accurately, but they did much better when they were shown how to use greaseproof paper. They reportedly had no difficulties analysing the symmetry of the basic shapes.

Posttests were identical to the pretests. The mean score on the written test was 82%; one child who scored 0% on the pretest scored

98% on the posttest. On the oral test, all the children selected all four symmetrical figures. As a further consequence, the investigator reports that after the unit children were much more careful "to get things in proportion" when drawing objects such as flowers, people and trees.

Nelson In the pretest, all the 30 pupils in her Grade 4 class could select from three figures the one which "can be folded so that one side fits exactly on the other side", but none knew the term "symmetry" and none could draw in a mirror line. All could name a circle, but none knew the terms "diameter" and "centre"; only one pupil could name the various types of triangle. The mean score was 24%.

Teaching extended over three weeks and consisted of 12 lessons in three sections. In the first section, pupils used paper folding to distinguish symmetrical and asymmetrical figures, and used mirrors to draw symmetrical figures. In the second section, they examined the symmetry of the various types of triangles, the square, the rectangle and the circle, and listed the corresponding properties of sides and angles and of the centre and diameter of a circle. The third section dealt with the bisection of the line segments and angles, again done by paper folding.

On the posttest, which differed from the pretest in one small detail, the mean score was 84%; all pupils recorded a gain in excess of 50%. All the students could draw the mirror line of a given symmetrical figure, but children still had some problems completing an open symmetrical figure with an oblique mirror line. Furthermore, only 30% gave the names of all five basic shapes and only 37% identified the diameter and centre of a circle.

L. Lawrence This study involved 15 Grade 5 pupils. In the pretest, pupils were required to identify symmetrical figures and to draw mirror lines and mirror images. The mean score was 35%, but there was much evidence of guessing. No pupil used the tracing paper provided to construct accurate mirror images, but several children drew reasonable images freehand.

The teaching unit, 12 lessons over six weeks, sought to strengthen pupils' intuitive concept of symmetry. They tested figures for symmetry and constructed symmetrical figures using paper folding, tearing and pricking, tracing paper, mirrors, and paint blobbing, but did not analyse the standard basic shapes for symmetry. Students were enthusiastic in carrying out the activities and often worked through break periods.

The mean score on the posttest was 54%. The investigator attributed the relatively small improvement to the fact that the questions on the test were read aloud on the pretest but not on the posttest. It is possible that children would have shown a greater improvement if the unit had included some more analytical activities.

Samuels The 20 children in this Grade 6 class had a mean score of 33% on a written pretest in which they had to draw and identify mirror lines and mirror images. Only 2 children drew reasonable mirror images.

In a teaching u nit extending over five weeks, pupils investigated symmetry using paper folding and pricking, mirrors, paint blobbing, charcoal imprinting, paper tearing and cutting, geoboard, tracing paper, flannel graph, graph paper and magic writing. They then investigated how the symmetry of a triangle depends on the number of congruent sides, and how many mirror lines various quadrilaterals have. The major difficulties were in reversing the orientation of mirror images and in drawing images in oblique (and sometimes horizontal) lines. Samuels reports that the unit generated great enthusiasm and helped the pupils develop independent and cooperative work habits, respect for each other and the teacher, and a friendly classroom atmosphere.

The mean score on the posttest was 83%. The score of one pupil increased from 0% on the pretest to 96% on the posttest.

Angles and Rotational Symmetry

Three investigators taught ideas of angle as an amount of rotation: V. Lawrence in Grade 6, McLean in Grade 7 and Pusey in Grade 9. Pusey also introduced the idea of rotational symmetry.

V. Lawrence A group of 20 regular attenders was selected from a Grade 6 class and given an oral pretest of ten questions on angle concepts; the mean score was 10%. A contrast group of pupils from a higher stream had a mean score of 15% on this test.

The teaching unit consisted of 20 lessons spread over four weeks. The first section dealt with the idea of turning; pupils identified everyday examples of angles, including angles between points of the compass, and expressed them as fractions of a revolution. They then made a paper right-angle and used it to classify angles as acute or obtuse. In the third section dealing with congruent angles, pupils copied and compared angles using drawings, cut-outs and sticks. Finally, pupils used an improvised protractor made from greaseproof paper to measure angles to the nearest 45°. Pupils had some problems understanding the material presented, especially in using the protractor, but they were generally interested in the unit.

For the posttest, the pretest was extended to 15 questions and given as written tests over three days. The mean score in the group which had been taught the unit was 61%, the girls scoring an average of 29% higher than the boys; the contrast group's mean score was again 15%. Although the pretest and posttest were not identical, it seems clear that pupils had learned a great deal about angles.

McLean From a top-stream Grade 7 class in a new secondary school, eight students were selected for study. Their mean on a unit pre-

test was 48%. Most students said that an angle was a 3-sided thing, obviously confusing angle with triangle. Also, most students could identify a right angle, but only one knew its measure was 90°.

The unit was taught in one 60-minute lesson a week for eight weeks. It covered the concepts of angle as a corner and an amount of turning; classification of angles; measuring and drawing angles using a standard protractor; and the calculation of adjacent and complementary angles. Concrete examples of angles were made using books, parts of the body, rulers, straws and card strips, and extracted from the environment. Students showed a great deal of interest in the unit and increased their score to a mean of 93% on the posttest.

Pusey This study involved 20 Grade 9 new secondary school students. On the pretest, no student knew the term 'line of symmetry'; but after explanation, 73% could draw the mirror lines in a given rhombus, rectangle and isosceles triangle and 40% drew all the mirror lines in a rectangle and an equilateral triangle. Half the students measured at least 3 out of 4 angles correctly, but none could rotate a line through a given angle. The mean score was 18%.

The teaching unit was limited to five lessons. These dealt with ideas of rotation, including revolutions and the centre and direction of rotation; classification of angles; measuring angles with a protractor; rotational symmetry, including order; and methods for rotating a figure through a given angle.

The mean score on the posttest, in which the two angle questions were the same as on the pretest, was 48%. In particular, 75% gave correct measurements for all four angles, 65% gave the correct order of rotational symmetry for a rectangle, and 30% rotated at least five out of six lines correctly. It seems that this investigator tried to cover too many ideas in the time available.

Solid Shapes

Two studies investigated the teaching of solid shapes: Martin in Grade 3 and McLeod in Grade 8.

Martin In the oral pretest given to the 24 pupils of this Grade 3 class, the term "solid" was not known to anyone. No pupil knew the mathematical names for a cube, cuboid, cylinder or cone from their models, preferring instead names of similarly-shaped real objects or the names of their faces. Students seemed to guess the number of faces, edges and vertices of the various solids. The mean score was 21%.

The teaching unit consisted of seven lessons over four weeks. Children sorted model solids according to their shapes; counted their

faces, edges and vertices; learned their mathematical names and found examples of each in their environment; studied the shape of the faces by copying them onto paper; and learned how to make the various models from card templates. Pupil interest was high throughout the unit.

Some interesting verbal difficulties surfaced during the teaching of this unit. At first, students said that a cube had only one face - but it also had a back, two sides, a top and a bottom. Corners and edges were also confused initially. Most interesting was a tendency to call a square a "cube-face" after it was discovered that all cubes have square faces.

The mean score on the posttest (similar to the pretest, but written) was 82%. Almost all pupils knew the names of the four solids discussed, and averaged 7 or 8 correct out of 10 questions on their numbers of faces, edges and vertices. Additional questions on the posttest showed that most pupils could also name the shapes of the various faces. Pupils also asked to draw the four solids; Figure 3 shows the most common types of drawing made.

McLeod This was the only study carried out in a technical high school; the 43 Grade 8 students involved had an average age typical of Grade 9 students in new secondary schools.

On the pretest, all students could recognise the basic plane shapes. Performance was moderate (35%-70%) on drawing and measuring lengths, drawing given figures accurately, stating the names of five common solid shapes, stating the numbers of faces, vertices and edges of a cube, and identifying the net of a cube. However, very few students (5% or less) could name a cubical object, make an accurate isometric drawing of a given model cube, or identify the nets of the other common solids. The overall mean score was 46%.

The teaching unit was contained in eight 80-minute lessons taught over 6 weeks. Methods of accurate drawing were revised first in order to ensure that students had the skills needed in the rest of the unit. The nets required to make a cube, cuboid, square pyramid, tetrahedron, cone and cylinder were then discussed in order. In each case, students predicted the shape of the net, discussed the dimensions to make a solid of a given size, drew the net accurately, and constructed the solid. Cardboard boxes were cut open and laid flat to assist in predicting the net of a cuboid, and a tin can was cut with shears to demonstrate the net of a cylinder. Students had the expected difficulty predicting the net of a cone, and the first models were all of very flat cones; but subsequent discussion brought out the relation between the shape of a cone and its net (see Figure 4). Two lessons dealt with the method of drawing cuboids in parallel projection, using both box and framework models.

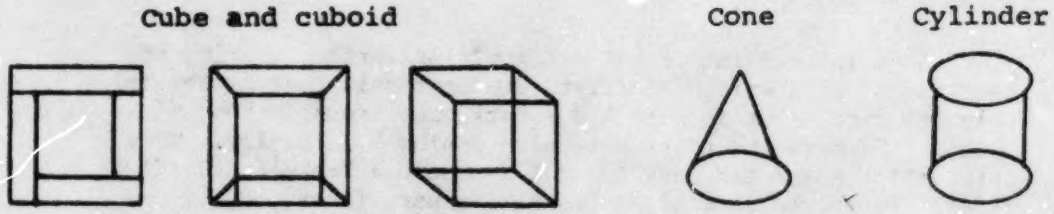

Figure 3 Most common methods Grade 3 pupils used to represent basic solids. From Martin's posttest.

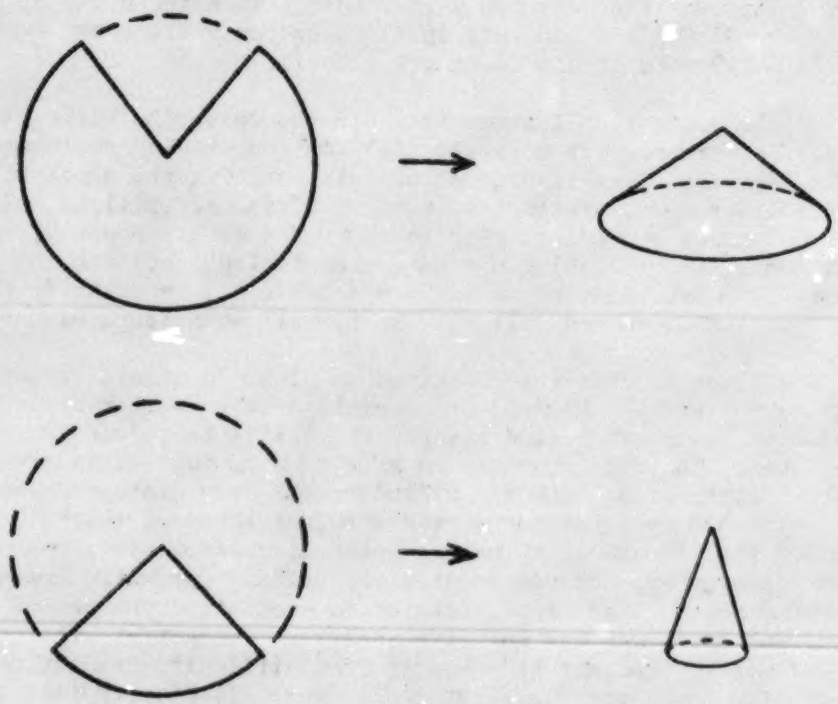

Figure 4 The relation between the shape of a cone and its net. From McLeod's Grade 8 teaching unit.

The investigator reported great interest in the model-making activities among both the boys and the girls in his class, although the boys were initially faster at drawing the nets. Considerable difficulties were experienced in learning to draw cuboids, and students were happy to return to model-making.

The posttest, which differed from the pretest in only a few minor details, showed a general improvement in accurate drawing skills and knowledge of solids; the mean score was 76%. Still only one student could name a cubical object and only 23% made an acceptable drawing of a cube; but cubical objects are in fact rather rare, and the investigator's criteria for assessing drawings of a cube may have been too high to detect conceptual gains.

Measurement

Two participants investigated measurement topics: West dealt with the circumference of a circle in Grade 8 and Largie taught metric measurement of length in a Grade 10 prefunctional group. To these might be added the studies of Webster, Anderson and Morrison (reported above under the heading "Fitting shapes together"), all of which included an introduction to area measurement.

West A group of 16 students were selected for study from a Grade 8 class in a new secondary school. On the pretest, all students knew that circles are drawn using compasses but none could give a precise definition of a circle. All could identify the centre of a circle, but only 31% knew the terms circumference and radius and none could identify a chord or an arc or state the relation between the lengths of a diameter and a radius. The mean score was 17%.

The teaching unit occupied six lessons and included revision of length measurement; the definition of a circle and a discussion of why certain objects are circular; the terminology associated with a circle; the relation between the lengths of a diameter and a radius; and the formula for the circumference of a circle (π times its diameter). The approach was through measurement activities, and the results were applied to familiar practical problems involving circles.

The mean score on the posttest was 29%, a rather disappointing result. The investigator reports problems with availability of geometrical instruments (only six for a class of 30) and several interruptions and absences due to an immanent Sports Day. Students also experienced difficulties manipulating the $C = \pi d$ formula in inverse calculations, a problem which there was no time to deal with. It also appears that the unit (and the unit test) laid too much emphasis on verbal definitions and did not give students enough time to practice using the relations discovered.

Largie This study concerned a group of 13 prefunctional (remedial) boys from a Grade 10 class in a new secondary school. The pretest, with a mean score of 19%, showed that most of the students could not select appropriate units to measure given lengths, estimate lengths in centimetres or metres, or draw or measure line segments in centimetres or millimetres. By contrast, a group of the best 13 continuing students in Grade 10 had a mean score of 86% on this test.

The teaching unit was given in eight 80-minute lessons taught over a one month period and dealt with measuring in non-standard units and the need for standard units; estimating and measuring lengths in metres, using strings and metre rules; estimating and measuring shorter lengths in centimetres, using centimetre strips of card gradulated by the students; drawing and measuring in millimetres, using an ordinary ruler, drawing triangles and rectangles with sides of given lengths; and conversion of length measurements between metres, centimetres and metres.

The students were very interested in the various measurement activities, and worked well in small groups. The use of non-standard units in the introduction seemed to have successfully taught the need for standard units, the need for units of different sizes, and the approximate nature of measurement. No difficulties were encountered using the improvised measuring instruments. However, the units centimetre and millimetre were often confused, and students' poor knowledge of decimals made conversion from one unit to another rather difficult.

The posttest was changed to make it more difficult for the contrast group, but covered essentially the same topics as the pretest. The prefunctional students had a mean score of 57% on this test, compared to 63% in the continuing group. The prefunctional students were still unable to convert measurements from one unit to another, but had clearly learnt a great deal about the practical aspects of measurement.

3. SUMMARY OF CLASSROOM FINDINGS

In this chapter, we shall attempt to summarise the findings of the 20 studies and draw some implications for teaching. We shall do this on the bold assumption that, despite the shortcomings of individual studies, the overall picture which evolves is probably indicative of how much Jamaican children of various ages know about geometrical concepts, how much they can learn, the problems they and their teachers are likely to face, and the rewards which can be derived from a greater emphasis on geometry in the school curriculum.

Children's present knowledge of geometry

The pretests given by the various investigators give a fairly clear picture of the average child's geometrical knowledge at present. Of the four basic plane shapes, the circle was recognized by all pupils tested; the triangle was known to 60-70% of Grade 2-3 pupils, and the rectangle and square were known to 20-60% of most Grade 2-6 classes, the actual percentage depending on the exact form of the test task. One participant suggested that one reason for this was that circles are much used in early workbooks and are usually referred to by name (eg "How many circles?" and "Circle the correct answer") but that squares and rectangles were usually referred to only as "boxes". Certainly children did not know much about circles; its centre was not identified by Grade 4 pupils, and the most frequent response in a Grade 8 class was that a circle was "a round thing used in Mathematics". Knowledge of properties of the other basic shapes (fitting together, parallel and perpendicular sides, symmetry) in Grade 4-6 was also very poor, as was knowledge of the angle concept in Grades 6 and 7. Grade 3 pupils knew very little about the basic solid shapes. These results are generally in agreement with the results of a recent survey of geometrical knowledge among Grade 5 and Grade 9 students in Kingston.[1]

There are several reasons to believe that this poor performance is simply the result of lack of exposure to geometrical ideas. Firstly, as will be seen from the teacher interviews summarised in the next chapter, geometry receives a very low priority in most teachers' yearly plans. Secondly, difficulties experienced by primary school children in drawing a line with a ruler, sticking squares onto paper, copying a figure using

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[1] Mitchelmore, M. C. Knowledge of basic geometrical concepts among Jamaican school children. Caribbean Journal of Education, 1982, 9, 14-31.

tracing paper, and many other constructive activities suggest that they have simply not had the manipulative experience with visual materials that young children in industrialised countries usually have in abundance in kindergarten or infant school or at home. Thirdly, poor performance was often due to the absence of technical vocabulary; when this was avoided or the meanings supplied (e.g. when testing children's concepts of symmetry), test scores jumped dramatically.

Children can learn much more geometry

The overwhelming evidence from the 20 teaching studies is that children can learn geometrical ideas if they are presented in an appropriate manner. The following table shows the changes recorded in those studies where the pretest and posttest were identical and the mean scores were reported.

Table 1 : Effectiveness of teaching studies

Investigator	Grade	Length of teaching unit	Change in mean score (%)
Lisle	3	(4 weeks)	70 (12 to 82)
Cunningham	4	(4 weeks)	42 (35 to 77)
James	4	(7 lessons)	70 (18 to 88)
Nelson	4	12 x 30 min.	60 (24 to 84)
Smith	5	7 x 50 min.	70 (22 to 92)
Anderson	6	15 x 60 min.	51 (8 to 59)
Douglas	6	6 x 60 min.	13 (43 to 54)
Samuels	6	(5 weeks)	50 (33 to 83)
McLean	7	8 x 60 min.	45 (48 to 93)
McLeod	8	8 x 80 min.	32 (46 to 78)
West	8	(6 lessons)	12 (17 to 29)
Pusey	9	(5 lessons)	30 (18 to 48)

In most of the 12 cases shown in Table 1, mean scores increased by at least 40%, a substantial gain by any standards. (The few studies which used a comparison group confirm that these gains could not have been simply retest effects). However, one cannot help noticing two studies with very small mean gains (12-13%) as well as two with quite modest gains (30-32%). What is the cause of these differences? The four units with the lowest gains

covered a variety of topics (parallels and perpendiculars, nets of solids, circles, and rotational symmetry) but most of them were relatively short (5-6 lessons) and dealt with relatively old children (Grades 8-9); perhaps the students were attempting to learn more advanced ideas without an adequate foundation, or perhaps they had already failed to learn the same material in earlier grades and reacted negatively to it. The two studies with the smallest gains could be faulted for using relatively few concrete activities (Douglas) and for emphasising verbal definitions (West). The four studies with the highest gains were undertaken with relatively young children (Grades 3-5) and placed great emphasis on children's making and examining shapes as well as their finding examples in the environment.

These findings support the view that geometrical ideas can and should be introduced informally in the primary school using an almost entirely concrete approach, and that if this is not done, ideas will be much more difficult to learn in secondary school when students are expected to reason in a semi-concrete or symbolic mode. However, concrete does not have to mean non-analytical. Although the youngest children are content to learn to recognise different shapes, parallels, symmetrical figures and so on, older children are certainly able to examine the various figures in more detail and derive geometrical relations distinguishing various classes of figure (e.g. number of faces, edges and vertices of solids, lengths or directions of sides of plane figures, symmetry of basic shapes). It would be as inefficient to deal only with overall shape recognition with children who can do some elementary analysis (as one or two studies in this project seemed to do) as it is to teach geometrical analysis to children who are not yet able to recognise overall shapes.

Some learning considerations

Motivation

Most of the investigators reported that children showed great interest in the topics taught, especially in the constructional activities. Pupils at times did not want to stop working on their projects, and often continued during break periods. They were eager to collect and bring objects to school to illustrate ideas discussed in class or to provide materials for the next investigation. They soaked up new ideas and were keen to learn new mathematical vocabulary. It also seemed to be quite easy for the teacher to organise small-group activities; although it must be remembered that the investigators were in most cases teaching smaller than average classes. Many investigators felt that their units

may have led to gains in :

(a) cooperation and mutual respect, since children often worked together towards a commonly-accepted goal;

(b) self-confidence, since even the "academically weak" child was able to achieve something of value;

(c) independent reasoning, since all statements were open to verification by appeal to concrete objects;

(d) aesthetic appreciation, since visual shapes and patterns were created and evaluated; and

(e) attitude towards school, since learning had become both pleasurable and stimulating.

Although arithmetic can probably be taught in such a way as to achieve similar gains, geometry seems to offer far more opportunities for the kind of small group and individual investigations which capture children's interest and imagination.

Linguistic difficulties

Many investigators reported that children had difficulties learning certain geometrical terms, even when they appeared to have no problem with the ideas involved. These difficulties can be traced to two factors:

(a) Familiar words are often used in mathematics in a technical sense which does not match their everyday meaning.

(b) Technical terms which children do not relate to words in everyday use may be confused because of their similar appearance or sound.

The first factor would explain why "face" was taken to mean "front surface" instead of "any distinct part of the surface"; also why "corner" (intended to be easier for children than "vertex") was taken to include the place where two walls meet, and so was confused with "edge". It could also explain why the square and rectangle were often confused, because both figures are "square" in the everyday sense of the word. In this sense, one class's preference for "cube-face" instead of "square" was eminently reasonable.

The second factor explains several further confusions which investigators reported:

rectangle-triangle-angle (very frequent)

intersect-bisect

millimetre-centimetre

Such confusions are likely to be greater among students of poor reading ability. By contrast, words like "parallel" and "perpendicular" seemed relatively easy to learn, even though (or perhaps because) they did not resemble any familiar word. One class reportedly preferred "perpendicular" to "right-angled", presumably because it was easier to distinguish from other "-angle" words. They might also have preferred "vertex" to "corner" if they had been given a chance.

How can teachers help children overcome such linguistic problems? In the first case, it is probably best for the teacher to show explicitly that the term has different meanings in mathematics and in everyday usage (neither of which is right or wrong); to explain why the mathematical usage is different, if there is any reason (e.g. the word may have two or more everyday meanings); and to give children plenty of practice in using the word in its geometrical sense.

In the second case, a specific effort should be made to link new unfamiliar words to similar, familiar words. The meanings of the prefixes tri-(three), inter-(between), bi-(two) and centi-(hundred) can be taught by forming "families" of words with these prefixes, including (for example) tricycle, international, bicycle and century. Pupils should be taught to recognize the prefixes by sight as well as by sound. This kind of activity will not only enable children to select the correct technical term from similar-sounding but rarely-used mathematical terms, but will also facilitate the learning of new words in other contexts (e.g. tripod, interchange, bilateral, centilitre). In this sense mathematics teaching must also include reading and language teaching; O'Connor[1] has confirmed the effectivenes of this approach in improving achievement in Grade 7.

Classification

It is an effective teaching technique to ask children to classify shapes and then to state the distinguishing characteristics of each class. But several studies showed that this procedure is likely to lead to some unexpected results, especially in young children. Thus children might initially put into different categories fat and thin rectangles (Golding), thick and thin cuboids (Martin), right-angled and other triangles (Golding), or large and small squares (Wedderburn). (Similar observations have been reported elsewhere.[2])

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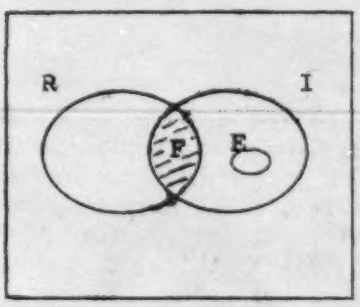
[1] An investigation of verbal problem solving in geometry .... Unpublished B.Ed. study, U.W.I. School of Education, Mona, 1975.

[2] Vollrath, H.J. The understanding of similarity and shape in classifying tasks. Educational Studies in Mathematics, 1977,8, 211-224.

These classifications are not wrong. By examining the characteristics which members of the two classes have in common and which distinguish them from members of other classes, and the characteristics which distinguish members of the two classes, the teacher can lead children to see that the two classes have more similarities than differences, so that they can reasonably be combined into one class and given a common name (using adjectives to distinguish two subclasses whenever necessary).

However, it must not be expected that adjectives will necessarily define subsets in children's minds. In particular, several investigators reported behaviour suggesting that children in Grades 4-6 do not regard a right-angled triangle as a triangle. (For example, asked to point to all the triangles in a set of shapes, children omitted the right-angled ones; but asked to point to the right-angled triangles, they made no errors.) This result may illustrate Piaget's finding that children at the early concrete operational stage do not fully understand the relation between a set and a subset defined by a special property, classically beads and brown beads[1]; or it may be that all the triangles which the teacher (and the textbook) have referred to by name have been approximately equilateral. The confusion may also have arisen from reliance on the sound of words; after all, if a triangle is not an angle, why should a right-angled triangle be a triangle? Here again it may be necessary for the primary teacher to show that right-angled triangles and other triangles have an important property in common and so can be given a common name. The various subsets of special triangles (right-angled, equilateral, isosceles) can then be examined and their special properties listed. Unfortunately, the Venn diagram is rather complicated so it is probably best left until early secondary school (see Figure 5).



U = {triangles}

R = {right-angled triangles}

I = {isosceles triangles}

E = {equilateral triangles}

F = {90°-45°-45° triangles}

Figure 5. Logical relation between various sets of triangles.

The primary teacher can still valuably emphasise that an adjective applied to any noun defines a subset, and can lead students to find further examples in other subject areas (e.g. "people" and "Jamaican people"). This is more language teaching!

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[1] Inhelder, B. & Piaget, J. The growth of logical thinking from childhood to adolescence. New York: Basic Books.

Learning about space

In addition to learning the names of various geometrical shapes and relations and various logical properties of these concepts, all of which can be described as verbal learning, it is clear that as a result of the units taught (especially in the primary grades) children also developed their intuitive understanding of space in ways which they could not be expected to verbalize. We will now attempt to verbalize these gains as regards three inportant concepts which have emerged in the twenty studies: congruence, pattern and direction.

The concept which appeared over and over again, but which was rarely named explicitly, was that of congruence. This concept appears to be fundamental to our understanding of space, and it must be formed intuitively long before it is given a name and a verbal definition. Several studies showed this happening among primary school children.

The concept of congruence develops in response to situations where two or more figures need to be "exactly the same". Examples quoted in Chapter 2 show that young children may not see any need for figures to be congruent, or if they do, they may not know how to achieve congruence. This should not be surprising; a great deal of cognitive development may be characterized as learning what "the same" means in different contexts. The teaching units presented situations where congruence was seen to occur because two shapes could be fitted on top of each other (using congruent squares of card, making symmetrical figures by paper folding etc.) and where congruence was seen to be important in order to produce a desired effect (pattern continuation, sticking squares to fit onto a square grid, copying a shape made on a geoboard, making a symmetrical figure). Through these activities, students must have become aware of need in certain circumstances for one shape to be an exact copy of another shape to the extent that one shape would, if moved, fit exactly on top of the other. Some activities also provided pupils with methods of copying shapes exactly in order to achieve congruence (using tracing paper, a squared grid, or measurement). Several studies showed that children learned to copy shapes fairly well when it was necessary, although they did not even try to do so before the unit was taught.

In examining examples of children's work included with the study reports, it would be easy to attribute their errors in copying to carelessness or lack of manipulative skill. There was certainly some element of the latter, but it cannot explain the extent of the errors which were made. And we do not believe that children engaged in a pleasing and interesting activity would do anything less than the best that they knew. It appears much more reasonable to assume that errors arose either because children did not realise that they should try to draw a congruent figure or because they did not know how to do so.

A second concept which turned up in several studies was that of covering a plane with a regular pattern. The basic idea, of repeating a given shape in some regular fashion, was soon grasped and led to much enjoyable activity of an aesthetic nature. At first, children did not fully appreciate the necessity for copying the given shape exactly or fitting a given copy exactly onto a square grid but as we have seen, the experience probably helped the development of their concept of congruence. The next step was the tessellation, a special kind of pattern in which the repeated copies of the given shape fit together side by side and leave no gaps. Making tessellations also proved to be an enjoyable aesthetic experience; in addition to providing further opportunities to experience congruence, it also drew attention to the necessity to orient copies appropriately (cf. Figure 2). The most important tessellation is no doubt the square grid, which embodies several critical properties of squares and rectangles and of parallels and perpendicular lines and is basic to the ideas of rectangular coordinates and area measurement. The studies of Anderson and Morrison showed that the idea of the square grid does not come naturally and needs to be supported by many practical exercises in making tessellations and experience of using squared paper in other geometrical activities.

A third important spatial concept is that of direction, which is closely related to the concept of an angle. Children did not seem to have any difficulties learning the ideas of parallel and perpendicular lines and of a right angle, all of which are rather static things. However, we have seen the difficulty which young children had in keeping copies of shapes in the correct orientation; and despite the use of copious real-life examples and even a simplified protractor in one case, two of the three investigators who taught units on angles reported that students experienced considerable difficulty learning how to measure angles. The major problems here seemed to be the concept of direction and the definition of angle in terms of turning from one direction to another. It would appear that children need many more concrete experiences involving these ideas before they can deal successfully with the semi-concrete angles which occur in drawings.

There are no doubt many other spatial concepts which can be developed through concrete geometrical experiences. (An important one, the relation between a solid figure and its possible representations in a drawing, was touched on by Martin and McLeod). The lesson of the studies in this project is that the average Jamaican primary school child has not had these experiences at home or at play and so has not been able to form the geometrical concepts which are implicitly taken for granted in standard secondary school geometry curricula. The primary school must therefore provide the child with myriad opportunities for cutting, sticking, copying, tracing, painting, weaving, drawing, model making and so on, if he or she is to stand a chance of making any sense of geometry later on. It is not enough to insert a unit on shape here and one on symmetry there; the concepts develop slowly and by abstraction from many different situations, so that activities should be frequent and varied.

Several studies show that a critical point in teaching is the transition from ideas developed with concrete materials to their representation in pictorial form (e.g. from covering a book with bottle caps to dividing a rectangle into squares, or from practical examples of turning to diagrams of angles). In the studies reported, children's difficulties were probably the result of attempting to proceed to pictorial representation too soon after the ideas were first introduced. The concrete experiences are very important, but it also seems important to leave children some time to assimilate the ideas in a concrete context before taking the next step.

Some teaching considerations

Classroom organization

We pointed above to the strongly motivational aspects of geometrical investigations, and the educational benefits to be derived. Needless to say, these benefits will not occur without proper planing and organization, in order to ensure that small groups can work together and work effectively. Materials must be assembled and prepared in advance, or children warned to bring materials. Expeditions outside the classroom may need the principal's permission. The teacher must decide whether to set the same task to all groups or to vary the task from one group to another. Groups must be formed. Instructions must be clearly given. Results must be collated and discussed in the large group and the main points summarised.

Nothing significantly new is involved. Twenty third year students managed to organise their classes according, but could a full-time teacher with a full-size class do so? He or she would probably need a lot of help, at least detailed notes describing possible activities and their objectives.

Materials and equipment

Few schools have money to buy materials for use in mathematics classes, and few students outside the high schools seem to have money to buy geometrical instruments. As we shall see in Chapter 4, these facts are often given as reasons for not teaching geometry. This project shows that these reasons are only excuses: concrete activities can be organised without commercially produced materials.

The most versatile concrete material is paper and card. Paper can be used to draw on, cut up, fold, paint, prick, weave. Paper can also be used for cutting shapes from, or for cutting strips to make shapes from, or for making models of solid shapes - but card is better. Providing one side is blank, scrap paper or card is as good as new: old circular letters, old posters, and the covers of used exercise books are obvious sources. The thoughtful teacher collects such materials throughout the year.

Geometrical instruments can also be improvised. A ruler can be made by hand from a strip of card, or lengths can be compared using pieces of string. A twice-folded piece of paper makes an adequate right-angle to replace the set square. A crude protractor can be drawn on greaseproof or tracing paper. A piece of string, or a strip of card with two holes in it, gives an adequate circle. In fact, these "instruments" embody the corresponding geometrical ideas much more clearly than commercial instruments do.

Other costless materials used included cloth scraps and bottlecaps, and hand mirrors brought from home. Straws and tracing paper (thin typing paper) are more expensive and should be used sparingly. Squared paper is even more expensive, but it is so valuable that it is worth the expense; a grid of 1 cm squares ruled on a gestetner stencil and duplicated is a preferable substitute for many activities, but it has to be done very carefully to avoid tearing the stencil. A pot of glue and at least one pair of scissors are essential equipment. A 9-pin geoboard, used with rubber bands, is easily made, and a Grade 7 woodwork class might be persuaded to make a class set as an elementary exercise. A flannel board is a luxury. Some materials (crayons, paint, charcoal) might be available through the art and craft grant.

Integration with other subjects

Several investigators integrated their units with art and craft lessons. This is as it should be, since art and geometry may, at the elementary level, be seen as two ways of reacting to our visual environment. The link between the two subjects reinforces both: the art and craft set the geometrical ideas in a concrete and satisfactory context, and the geometry provides a vocabulary for talking about the artistic patterns and the craft designs. It is unfortunate that art and craft teaching does not currently make the link with geometry; for example, "paint blobbing" may take place without any mention of symmetry. Several studies in this project show how easily the link can be made, and how valuable it can be.

Links with reading and language teaching have been mentioned. New words introduced in geometry lessons can be examined in the next language arts lesson, to form "word families". After a particularly interesting geometrical investigation, children can be asked to describe what they found or how they found it. Written instructions for small group geometry activities can use language structures recently discussed and give practice in reading comprehension. There are many opportunities for similar integration between mathematics (not only geometry) and language teaching.

But perhaps the most important link that should be emphasized is between geometry and arithmetic. One investigator (Morrison) found that Grade 6 children did not attempt to use multiplication

to count the number of squares in a rectangle, suggesting that children had a limited knowledge of the practical utility of multiplication as a problem-solving tool; see how much more powerful their concept of multiplication would have been, and how much easier area measurement would have been, if these children had always thought of multiplication in terms of rectangular arrays (see Figure 6).

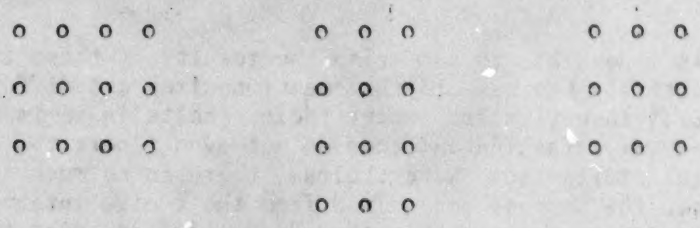

Figure 6. Rectangular arrays illustrating 3x4, 4x3 and 3x3.

Such examples show the commutativity of multiplication as well as the difference between rectangles and squares, and even illustrate symmetry and rotation. Another important visual aid to understanding arithmetic is the number line; the equal spacing of the integers, and the equal division of the intervals to create fractions, provide further experiences to develop the concept of congruence which, we argued above, is so critical and so much in need of strengthening. There are many other examples in which spatial situations can be used to provide powerful models of arithmetical operations or interesting applications, and at the same time develop children's spatial intuition and reinforce their geometrical knowledge.

Several places in which geometrical topics arise or could be made to arise in other subjects on the curriculum were suggested by the participants in this project; making regular and symmetrical shapes in Dance and Physical Education, shapes and measurement in Science (e.g. comparing the shape and area of leaves and flowers), Geography (where various imaginery lines occur), and Home Economics (shapes of pots, patterns in needlework), and so on. The primary school teacher is in an ideal position to exploit these opportunities by either drawing attention to the geometrical ideas as they occur in other subjects or by refering to these occurences when introducing new ideas in geometry lessons.

4. INTERVIEWS WITH PRIMARY SCHOOL TEACHERS

Most investigators interviewed their fellow teachers in an attempt to find reasons for children's poor performance in geometry. Eliminating two studies done in secondary schools, which have specialist mathematics teachers, we are left with twelve which deal with the teaching of geometry in primary schools, namely the studies of Anderson, Cunningham, Golding, James, V. Lawrence, Lisle, Martin, Nelson, Samuels, Smith, Webster and Wedderburn.

It is impossible to summarise the results of these interviews objectively because all the questionnaires are different; also, several investigators report their results in an impressionistic, discursive fashion and some do not even report the numbers of teachers interviewed. Nevertheless, there is so much agreement between the impressions gained from the twelve interviews that one is led to believe that they do validly describe the actual situation in most of our primary schools. A subjective summary will therefore be attempted, quoting figures where they are available, if only to suggest areas for more objective study.

The amount of geometry teaching

It is obvious that the main reason why children do not do better in geometry is that the subject is seriously neglected in primary schools. All investigators, including those who did not carry out formal interviews, report gaining the impression during their internship that very little geometry is taught. In two schools, one teacher in each of Grades 1 to 4 was interviewed; 7 stated that they never taught any geometry and the eighth taught only recognition of the circle, square, rectangle and triangle (in Grade 1). In another school, only one out of 4 teachers admitted to teaching "a little" geometry. All 10 teachers at another school stated that less than 20% of the mathematics curriculum for their grade dealt with geometry, but it is not clear whether answers referred to the syllabus or to the teaching which took place. However, 5 out of 8 teachers interviewed at another school denied that they avoided teaching geometry wherever possible.

The main reason why so little geometry is taught is probably the fact that teachers do not like or enjoy teaching geometry. Five investigators asked teachers to say which they preferred teaching: arithmetic, algebra or geometry; out of a total of 70 teachers interviewed, only 8 put geometry at the top of the list. Despite the fact that most teachers enjoy teaching mathematics as a whole (9 out of 10 in one school), it seems that the teaching of geometry is generally disliked (by 75% of the teachers at two other schools).

Reasons given for not liking to teach geometry (or for not teaching geometry, which probably amounts to the same thing in practice) can be divided into two categories, one relating to teachers' assessment of their competence and the other to teachers' perceptions of geometry as a subject.

Teachers' knowledge of geometry

Most of the teachers at two schools cited lack of competence as a reason for not teaching geometry. At a third school, only 5 out of 10 teachers said they were competent to teach geometry; at another, 5 out of 8 teachers said they would need more help before they could teach their own children geometry. All 6 teachers at a fifth school confessed that they did not know enough about methods of teaching geometry. All 4 teachers at another school said they would not teach symmetry if asked because they did not know enough about it and had not done any geometry since leaving college.

It appears that most teachers were, in their time as students, the victims of the same problems which we are currently investigating. The teachers at one school pointed out that most of them had started teaching with only a Jamaican School Certificate, which did not initially include much geometry; the Third Jamaica Local Examination, the predecessor of the JSC, included no geometry at all. Furthermore, geometry has only been included in teachers' college programmes in the past ten years.

As a result, teachers clearly do not feel confident teaching geometry. Only one out of ten teachers at one school felt confident teaching the subject. At another school, none of eight teachers gave geometry as the area of mathematics that they felt most comfortable teaching.

Teachers' perception of geometry

Teachers' dislike for teaching geometry also stems from their dislike of geometry itself. Although 5 out of 7 teachers at one school indicated that they liked mathematics in general, 75% of the teachers at another school and most of the teachers at a third answered "no" to the question "Do you like geometry"?

Geometry is clearly perceived as a difficult and dull subject. Most of the teachers at one school described geometry as a "hard" subject, and none of the 10 teachers at another school agreed that geometry was easy to learn or that pupils enjoyed learning geometry. Most of the teachers at one school said that they disliked geometry because it was complicated and uninteresting.

Teachers also believe that geometry is of little practical use. At different schools, few teachers agreed with statements that geometric ideas prepared pupils for life (3 out of 10), that it was important for its utility value (2 out of 10), that layman needed a knowledge of geometry (2 out of 10), or even that tradesmen made use of geome-

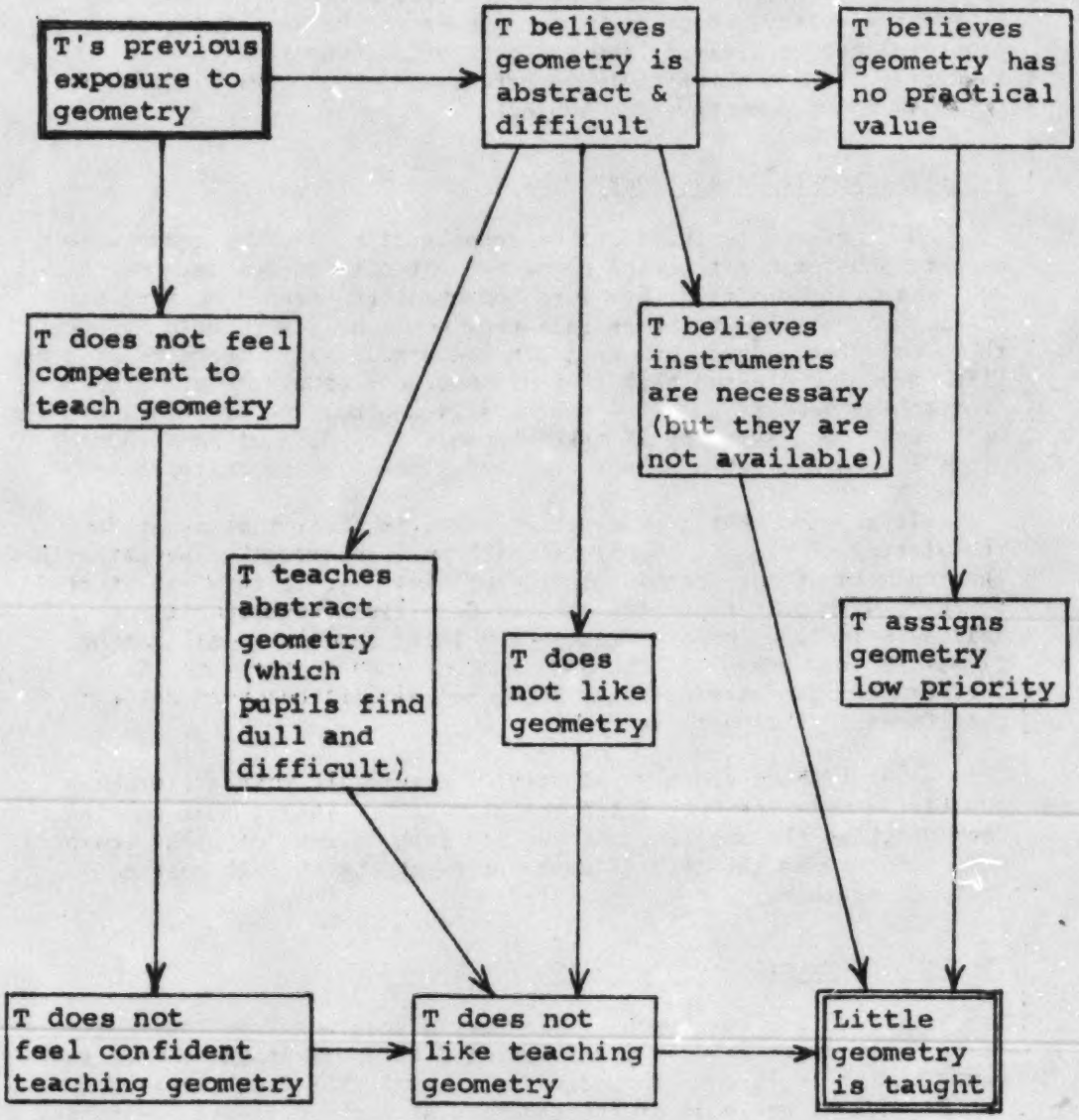
T's previous exposure to geometry
T believes geometry is abstract & difficult
T believes geometry has no practical value
T does not feel competent to teach geometry
T believes instruments are necessary (but they are not available)
T teaches abstract geometry (which pupils find dull and difficult)
T does not like geometry
T assigns geometry low priority
T does not feel confident teaching geometry
T does not like teaching geometry
Little geometry is taught

Figure 7 Proposed paths by which the previous exposure of a teacher (T) to geometry as an abstract, dull and difficult subject causes little geometry to be taught in primary schools.

trical ideas (5 out of 10). However, at another school none of the 8 teachers agreed that pupils would have no use for geometry when they left primary school and only 2 agreed that working class people do not find geometry useful for everyday life.

These views of geometry are so different to those of the participants in the Project and their pupils that one is forced to wonder whether the teachers were thinking of the same subject as the interviewers. The answer is probably no: "geometry" to most primary school teachers seems to mean traditional, abstract, euclidean geometry. This impression is strengthened by the frequency with which teachers so often gave lack of geometrical instruments as the main reason why geometry is not taught (five out of seven teachers at one school, all six teachers at another school) or why students perform poorly at geometry (six out of of eight teachers at one school, six out of ten at another). The instruments would presumably be needed for abstract constructions in the euclidean tradition.

Given this perception of geometry, it is hardly surprising that many teachers said that it was of little importance in primary school mathematics. At one school, 5 out of 6 teachers ranked geometry after arithmetic and algebra in order of importance, and most of the teachers at two other schools said that pupils did not need to learn geometry in the primary school. At two schools, teachers were unanimous that geometry was less important than the "Four Rules" and that backward children would be better occupied learning their addition and multiplication facts. In other schools, most teachers stated that other areas of mathematics had greater priority and that geometry should be left until high school. All is not lost, however. At three schools, most or all of the teachers said that it was necessary to have some geometry on the primary school curriculum, and at one of these, 7 out of 8 teachers agreed that geometry was as important as other areas of mathematics.

Summary

The paths by which various factors seem to influence a teacher to decide not to teach geometry are illustrated in Figure 7. Unfortunately, it is actually a vicious circle, because the next generation of primary school teachers will have had little exposure to geometry and will probably also perceive it as an abstract, difficult subject.

Where can the vicious circle be broken? The obvious point is at the teachers' colleges, where the importance of teaching elementary geometrical ideas through concrete materials has been recognised for some time now; the individual studies done in this Project show that it is possible to graduate new teachers who like geometry, who like teaching it, and who are well able to teach children to learn and

to like geometry. For teachers already in service, it is to be hoped that the hard-pressed Ministry of Education will be able to find time in their programme of seminars to persuade teachers that primary school geometry can be the very opposite of the abstract, difficult and dull subject which they experienced at school.

APPENDIX A.

JAMAICAN STUDENTS' KNOWLEDGE OF GEOMETRY

Jamaican students generally attain a disappointingly low level of achievement in mathematics, whether one thinks in absolute terms (12)* or in comparison to other West Indian students (14). Their performance in geometry is a matter of particular concern.

The purpose of this paper is to review what is known on Jamaican students' knowledge of geometry, in order to throw light on the nature of students' difficulties, to guess at how they have come about and to propose measures for their alleviation.

PERFORMANCE IN GEOMETRY

Evidence from Public Examinations

Item analyses of the Grade Nine Achievement Test (GNAT) and the Jamaica School Certificate (JSC) and Secondary School Certificate (SSC) examinations give a picture of the average level of achievement of Jamaican students at or near the end of their formal schooling, except for the 10% or so who are selected for the academic High Schools. The GNAT is taken by selected All-Age School students at the end of Grade 9; the JSC is an open examination at about Grade 10 level; and the SSC is taken by all students in the (New) Secondary Schools at the end of Grade 11. All items on the examination papers are multiple-choice. Here are some recent results, made available by courtesy of the Ministry of Education, Kingston:

35% classified 150° as an obtuse angle (GNAT 1975)
35% recognised the line of symmetry of a figure (GNAT 1977)
38% identified diagrams of a pyramid, cylinder and cube (GNAT 1977)
21% knew the number of faces of a cone (JSC 1976)
32% selected two congruent triangles from a set of four (JSC 1973)
59% recognised a diagram of a hexagon (SSC 1978)
29% knew the number of faces of a square pyramid (SSC 1978).

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*Numbers refer to the references on pages 53-55.

Performance on related measurement items has also been rather poor. For example:

14% found the area of a figure on a grid (GNAT 1977)
30% calculated the area of a dimensioned triangle (JSC 1973)
23% measured the length of a line to the nearest millimetre (SSC 1978).

On the other hand, calculations involving angles seem to be rather easier. For example:

60% calculated the second angle on a straight line (GNAT 1977)
54% calculated the third angle of a triangle (SSC 1978).

It is also noticeable on the item analyses that the lowest third of the students tend to choose the four or five alternatives in roughly equal proportions, suggesting that they are guessing at random.

It has not been possible to find similar examination data for High School students.

Evidence from Other Tests

A few studies carried out in the School of Education at Mona have included geometry tests which have been analysed by topic. These give further information on student achievement.

O'Connor (21) surveyed a random sample of classes from three New Secondary Schools (urban and rural) at the end of the first term in Grade 7. She found that:

5% knew the terms angle bisector, congruent figures and quadrilateral
10% calculated the third angle of a triangle
20% recognised a diagram of a right angle.

Noble (20) tested a mixed-ability Grade 7 class in an urban New Secondary School on various measurement skills. She found that:

25% measured a 5cm line correctly
None drew a 5cm line accurately
None could read a scale when the pointer was half-way between two labelled marks.

These figures suggest that the average student entering a non-selective secondary school is probably totally ignorant of geometry.

Some students do learn some geometry in secondary school. Holmes (11) found in two top-stream Grade 9 classes of an urban school that:

98% recognised a diagram of a square
66% recognised a diagram of a right angle.

However, only

7% recognised a diagram of a parallelogram.

Even amongst the best students, it would seem that knowledge is very limited.

Summary of Test Results

Assuming that the tests cited validly measure students' knowledge, we conclude that Jamaican students who are not selected to enter the academic High Schools are almost totally ignorant of geometrical concepts at the start of Grade 7. Some geometry is learned in Grades 7-11, but at the end of Grade 11, only about one-third of the students possess such basic knowledge as the mathematical names of common 2- and 3-dimensional shapes, the meaning of such concepts as congruence and symmetry, or methods for measuring lengths or calculating non-rectangular areas.

CHILDREN LEARNING GEOMETRY

Test and examination results confirm the existence of the problem, but they do little to help us understand it. Why do Jamaican students find it so difficult to learn geometry? To illuminate this question, we shall look next at some anecdotal evidence taken from reports of geometry lessons taught on teaching practice by students in the School of Education at Mona. Unless otherwise stated, the studies were carried out in New Secondary Schools.

Grade 7 Students

Two recent students (Adams (1) and Davis (7))taught units on elementary solid geometry. Both students had their pupils

sorting solids and learning their names; counting faces, edges and vertices; identifying the shapes of the faces; and distinguishing plane and curved edges and faces. Davis' pupils also looked for parallel and perpendicular edges and faces, and made model cuboids. Although Adams' class was a remedial group and Davis' class was a top-stream group, they reported similar problems:

1) Names for 2- and 3- dimensional figures were confused; cubes were called squares and circles spheres.

2) The abstract concept of a point, and therefore vertex, was not known at first. Also, students did not know how to represent and label a point in a diagram.

3) Asked to identify a line or an edge, students were often content at locating the end-points and ignored everything in-between. A line AB was called "A and B".

Davis' students also had difficulty reasoning from a skeletonic diagram of a cuboid.

Two further studies examined the teaching of angle concepts. Noble (20) started by discussing concepts of fitting and turning. Students' initial ideas of fitting seemed to allow for slack and protrusion, but they soon learned the concept of a geometrically exact fit. Practical experience of turning was much more familiar, but students found it difficult to calculate the amount of turning in an angle diagram as a fraction of a revolution. Students learned to use a paper right-angle to identify acute and obtuse angles, but most of them failed to learn how to use a protractor through confusing the two scales. Reid-Waugh (23), teaching a mixed ability High School class, also reported difficulty distinguishing the two scales on a protractor. She also found evidence of poor understanding of the concept of angle; students said that the paper angles torn from a triangle and fitted together as shown in Figure 8 formed a semicircle (instead of a straight line).

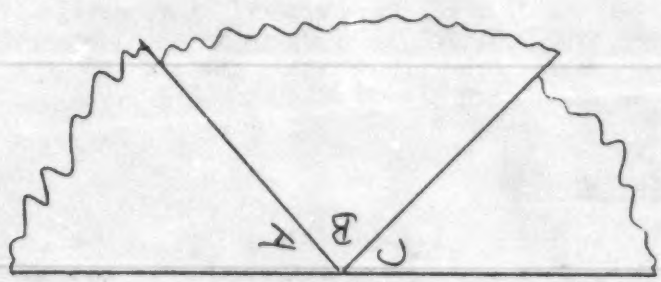

Figure 8

The similarity between the experiences of Noble and Reid-Waugh suggests that High School students also enter Grade 7 with very little knowledge of geometry.

Grade 9 Students

Two students (Duncan (8) and Shippey (27)) taught symmetry, reflection and translation to top-stream classes using repeating patterns as an introduction. Once again, both report similar difficulties. At first, some students did not understand how to continue a given pattern, others drew arbitary shapes or were careless in copying the base figure - none used the tracing paper provided; they all had difficulty explaining how they had continued the pattern. However, performance improved substantially through structured practice on simple patterns. Students were enthusiastic about mirror symmetry, and much more accurate in their drawings. Shippey also reported good student work making tessellations from cardboard cut-outs, and drawing and identifying translations on a grid using column vectors.

The Nature of Students' Difficulties

From the studies reviewed (admittedly limited, but supported by my own informal observations and discussion with colleagues), the nature of students' difficulties becomes clear: secondary level geometry builds on concepts which many entering students do not understand. Concepts as basic as point, line and angle have not yet been abstracted from their experience of physical space, and 2- and 3-dimensional space are not clearly differentiated. Most fundamentally, students apparently do not see any necessity to copy or fit shapes accurately, so there is no basis for the concept of congruence. It is no wonder that students find geometry so difficult.

This conclusion is supported by the results of a study I did recently to compare the spatial ability of samples of children aged 7-15 years in Kingston, Jamaica; Columbus, Ohio; and Bristol, England (19). "Spatial ability" is the ability to form and manipulate mental images of figures in space; it requires one to copy or draw a given figure, or to recognise it when it is embedded in a distracting background or turned over, turned around, folded up, cut apart, or otherwise transformed - without necessarily being able to name or describe the figure in words. Spatial ability is clearly closely related to the comprehension of the basic geometrical concepts just mentioned; in my study, it was measured by a pattern-copying task and a test of 3D drawing. I found that the Jamaican children were (an average) three years behind the American children and six years behind the English

children at each age level. And yet Jamaican children attempt roughly the same secondary geometry curriculum as English children!

THE TEACHERS OF GEOMETRY

One obvious reason why Jamaican students do not learn the fundamental geometrical concepts needed for secondary school geometry is that very little geometry is taught in the primary schools. I have no objective evidence for this conclusion, but the Ministry of Education is so convinced of its truth that it will allow no geometry items on the 11+ (Common Entrance) examination; such items, they argue, would given an unfair advantage to students from schools where geometry is taught. Another indicator is the frequency of reports that students on teaching practice have been asked to teach the geometry topics on the syllabus, which would, we infer, otherwise have been omitted.

It is amusing to note similar comments coming from even highly industrialised countries. Here are some quotes from the February 1979 issue of the Arithmetic Teacher, published in the U. S. A.

> The teaching of geometric concepts is sporadic (p.10).
> Geometry is important. It should be taught (p.14).
> Geometry activities usually are considered a "frill", a topic that is covered if time permits (p.17).

Wirzup (29) reports that the primary school mathematics curriculum in the USSR has been redesigned within the past ten years to remediate a similar lack of emphasis on geometry in that country. The number of "effective geometry problems" in the standard textbook has been increased from 1.2% to 18.5% of all exercises!

Teachers' Knowledge of Geometry

The apparent reason why geometry is not taught in Jamaican primary schools is that the teachers are unsure of the subject themselves. This should not be surprising, because they are the products of the very system in which geometry achievement is so poor.

There is little objective evidence on the level of teachers' knowledge of geometry, however, In 1972, I gave a group of high and secondary school mathematics teachers a set of five spatial

ability tests, and then compared their performance with that of a group of student teachers in the U.S.A.; the U.S. students were markedly superior or all tests (16). More recently, I interviewed a small but representative group of prospective primary teachers in their first year at a college in Kingston (18). I summed up my findings thus:

> Most students could identify angles which were a multiple of 45° (provided one leg was horizontal), knew that the base angles of an isosceles triangle were equal (provided the base was horizontal) and knew the names circle, rectangle and square (provided the base of the square was horizontal). However....at least half the students were unable to deal with angles of turning; with a triangle or a square in a non-standard position; with the concepts of opposite sides, parallel sides, diagonals, mid-point, and the angle between two lines in a figure; or with the conventional notation for labelling figures (p. 7).

Once again, the picture presented is one of lack of basic geometrical concepts.

Since geometry is taught in Jamaican teachers' colleges, teachers now seem to be entering schools with much improved knowledge. For example, in the 1978 final examinations in mathematics for prospective primary and pre-primary teachers:

> 62% measured an angle to the nearest 5°
> 74% recognised the net of a cube
> 63% defined the term "similar" correctly
> 61% calculated an unknown side in a similar triangle.

These are not great results, but they are encouraging.

SUCCESSFUL TEACHING OF GEOMETRY

Are we to conclude from the evidence presented above that Jamaican teachers are incapable of teaching geometry, and that Jamaican students are incapable of learning the subject? By no means! The fact that a situation exists does not mean that it must or should continue to exist.

Firstly, there is clear proof that teachers can learn geometry and can learn to teach geometry. One research report (17) demonstrates that first and second year teachers at a teachers' college in Kingston successfully learned geometry using individualised workcards. We have already cited encourag-

ing examination results subsequent on a recent increase of emphasis on the teaching of geometry in the teachers' colleges. In the School of Education at Mona, nearly half the students taking the Certificate of Education programme in Mathematics over the past five years have chosen to make geometry in secondary schools the subject of their teaching project. Teachers can learn and teach geometry.

Secondly, it is clear from the reports of teaching projects undertaken at Mona that Grade 7 and Grade 9 pupils can also learn geometry. Some of the learning which has taken place was described in the section "Children Learning Geometry" above. The studies of Holmes (11), Leon (13), Noble (20), O'Connor (21) and Shippey (27) also show overall satisfactory performance on unit post-tests. Where performance has been less than satisfactory, it would seem to be a result of teachers' attempting to rush on too quickly after teaching a basic concept; for example, Noble's teaching the use of a protractor almost immediately after teaching the apparently difficult idea of an "amount of turning" (20).

A Model of Geometry Learning

We pause at this point to expound a model of geometry learning which has received little exposure but which is very appropriate to our discussion. The model is due to two Dutch educators, P.M. van Hiele and his wife D. van Hiele-Geldof. Our summary is taken from an article by Wirszup (29).

The van Hiele model proposes that geometrical knowledge passes through five levels. The levels are discontinuous in the sense that a child operating at one level cannot understand concepts at any higher level. He or she continues at one level until a "jump" to the next occurs. Thus, teaching effort is wasted unless it is pitched at the child's operating level. Briefly, the five levels are as follows:

Level 1

Figures are perceived as whole entities. Different 2- and 3-dimensional shapes are distinguished and can be reproduced, but on the basis of their overall shape not their intrinsic properties. Thus, a square is seen as different from a rectangle because it looks different. One might say that children at this level take a holistic approach to geometrical figures.

Level II

Different components of a figure are perceived, and the relations between them are established experimentally. A figure is now described by its properties, rather than its shape. However, squares and rectangles are still regarded as different, because they have different sets of properties. Children at this stage are beginning to show an analytical approach to geometrical figures.

Level III

Relations between properties are now perceived, and elementary deductive chains are formed. A square is now seen as a special type of rectangle. This stage marks the beginning of a logical approach to geometry.

Level IV

The role of definitions, axioms and theorems is understood, and deduction can proceed from various premises (e.g. alternative definitions of a square). Logic is fully developed, but still in application to concrete objects.

Level V

The system is completely abstracted from concrete geometrical objects, and deduction proceeds at the highest level of logical rigour. Geometry is now only one example of an axiomatic system.

In terms of this model, we can see that the geometry curriculum followed in secondary and high schools in the Caribbean assumes that children are initially operating at Level II, and it takes most children through Level III. (The old G.C.E. "Syllabus A" geometry used to extend through Level IV, but now-a-days Level IV teaching is not common until university. Level V is limited to undergraduate honours courses and beyond). If the van Hiele hypothesis is true, then it follows that the primary school curriculum should cover Level I and the beginning of Level II. The problem of poor geometry achievement among Jamaican children thus appears to be the result of a lack of Level I teaching in the primary schools; the provision of introductory Level I activities in the secondary school geometry unit intended to deal with Level II concepts is unlikely to be successful because there will not be time for the students to pass from Level I to level II thinking.

A Programme to Improve Geometry Achievement

The alleviation of Jamaican children's difficulties in learning geometry therefore lies entirely in the primary school mathematics curriculum. Many more activities need to be included which enable students to develop holistic Level I concepts well before they attempt to analyse geometric figures at Level II. Basic holistic concepts include in the following:

Types of 3-dimensional shapes, and their names
Types of 2-dimensional shapes, and their names
Spatial regularity, as in patterns
Balance (mirror symmetry)
Sameness of shape, independent of size (similarity)
Sameness of shape and size(congruence)

These concepts can be treated holistically or analytically, but it is important that only the holistic approach be taken initially. The student objectives would be to name given 2- or 3dimensional shapes, and to recognise when a pattern is regular, when a shape is balanced, and when two figures have the same overall shapes; the student would not be expected to give any reason other than "because that is what it looks like".

Some of the above concepts are already providing tools (symmetry, congruence) for the analysis of figures at Level II. Some other tools would arise later in Level I as children look more closely at various components of spatial configurations. These include:

Surfaces and regions
Lines and curves as edges of solids or boundaries of regions
Points as intersections or specific locations
Flatness and straightness
Turning; central and rotational symmetry
Angle as the region between two lines or an amount of turning
Right angles and perpendiculars
Parallel lines

At Level I, the emphasis in all these cases is still on what things look like as a whole. For example, children would be expected to recognise parallel lines drawn on a sheet of paper or in their environment, but would not be expected to define the term, to construct accurate parallels, or to name figures with parallel sides.

At Level II, the analysis begins. While still in primary school, children can investigate questions like these:

How many faces/edges/vertices does a cube have?
What shapes are the faces of a pyramid?
What solids have curved faces?
Are any faces of a cylinder the same (congruent)?
What are the angles of a square?
Is there any relation between the lengths of the sides of a rectangle?
What plane figures have parallel opposite sides?
What is the angle between North and East?
What parts of a symmetrical figure are congruent?
How are these parts related to each other?

These questions can only be answered if the component concepts have been learned at Level I. More complex relationships may be studied at secondary school.

Many of the activities through which concepts and relations are learned are very similar at Level I and Level II; it is only the questions and answers which differ. For example, a useful exercise is to have children bring various containers and other objects to school and sort them. At Level I, the sorting would concentrate on the overall shape and would lead to sets of cubes, cuboids, cylinders etc. Difinitions would be of the nature "a cuboid is a box shape". At Level II, children could also be asked to list ways in which any two of these are different, and would arrive at definitions such as "a cuboid is a solid with six rectangular faces, eight vertices and twelve edges; all the faces are rectangles".

Another useful exercise is drawing tessellations of congruent triangles. At Level I, the major result is that the trrangles fit together like tiles in a regular pattern leaving no gaps; children can then try various other shapes (rectangles, circles,...) and look for tessellations in their surroundings (block walls, textile prints,...). At Level II, it will be noticed that the tessellation contains three sets of parallel lines, that each vertex lies on one line from each set, that the three angles of the triangle occur twice at each vertex, and that the tessellation has central symmetry above each vertex; these observations lead to the angle sum property of a triangle, to properties of corresponding and alternate angles, and to various properties of parallelograms and enlargements. The important thing is that teachers should be aware of the level at which their children are operating and use the activity to teach concepts at that level only.

Some Difficulties to be Faced

There is no hiding the fact that Jamaican teachers who attempt to teach geometry in the manner suggested are likely to face a number of difficulties. First in many teachers' minds will be the

mathematics syllabus (15). Based on a text from the days of "new mathematics" in the U.S.A. (4), the syllabus puts an initial emphasis on logically basic concepts such as points, lines, paths and regions but treats these in isolation instead of abstracting the ideas from real-life experience; there is no three-dimensional work until Grade 5, where there is a short, abstract section on planes. In other words, both the sequence and the approach differ fundamentally from that proposed above.

Secondly, if geometrical concepts are formed by interacting with the environment, then teachers must supply structured activities for children. Telling will not do at all. But this brings three problems:

(1) Where are teachers to get ideas from? Many suggestions have been published in activity-oriented general methodology books (2, 3, 9, 24, 26), in books on the teaching of geometrical topics (5, 6, 22, 28), in teachers' magazines such as Mathematics Teaching and Mathematics in Schools, and in topic booklets written for students (see (10) and (25) for bibliographies), but in Jamaica these materials are not available for most teachers to consult.

(2) Where are teachers to get materials from? Children need at least paper and pencil, but even these are in short supply, especially in the rural areas. One would like to have scissors, instruments, cardboard, tracing paper, grid paper ....., but funds to buy such materials are just not there; one must make do. The determined teacher will conserve every possible resource (including cardboard boxes and tubes, cotton reels, scrap paper, and covers of used exercise books) and beg remorselessly (wooden offcuts, thin typing paper for tracing, etc).

(3) How can teachers organise activities in crowded classrooms? Teaching 60 children at once is difficult enough, but organising the same 60 to carry out individual or small group investigations is a colossal task. Also, many teachers (and principals) have problems distinguishing the buzz of goal-oriented activity from the noise of a class out of control.

At this moment, one can offer little advice to teachers on how to overcome these difficulties. Innovation is likely to be an uphill task until the Ministry of Education, the teachers' colleges, school principals and parents, realising the sorry state of Jamaican students' performance in mathematics in general and geometry in particular, all become sympathetic to efforts to improve the situation and give them their full support.

REFERENCES

(1) ADAMS, P. 1977: A Teaching Project in Geometry for a Group of Grade 7 Remedial Pupils in a Jamaican Secondary School. B.Ed. study, U.W.I. School of Education, Mona.

(2) ASSOCIATION OF TEACHERS OF MATHEMATICS, 1967: Notes on Mathematics in Primary Schools, C.U.P., Cambridge.

(3) BIGGS, E.E., & MACLEAN, J.R., 1969: Freedom to Learn: An Active Learning Approach to Mathematics. Addison-Wesley, Ontario.

(4) BROWNELL, W.A., et al, 1965: Mathematics We Need. Jamaican edition. Ginn, Boston, Mass.

(5) BRUNI, J.V., 1977: Experiencing Geometry. Wadsworth, Belmont, Calif.

(6) BRYDEGAARD, M., & INSKEER, J.E. Jr. (Eds.) 1970: Readings in Geometry from the Arithmetic Teacher. National Council of Teachers of Mathematics, Reston, Va.

(7) DAVIS, D.R., 1978: Teaching Geometry to Grade 7. Cert. Ed. study, U.W.I. School of Education, Mona.

(8) DUNCAN, A.M., 1979: A Survey of Some Problems Encountered in Teaching Relations and Transformation Geometry to two Grade 9 Classes in a (New) Secondary School in the Corporate Area. Dip. Ed. study, U.W.I. School of Education, Mona.

(9) FITZGERALD, W., et al, 1973: Laboratory Manual for Elementary Teachers. Prindle, New York.

(10) HARDGROVE, C.E., & MILLER, H.F., 1973: Mathematics Library: Elementary and Junior High School. National Council of Teachers of Mathematics, Reston, Va.

(11) HOLMES, N.L., 1978: A Transformation Approach to Geometry using the Guided Discovery Method. Cert. Ed. study, U.W.I. School of Education, Mona.

(12) ISAACS, I., 1975: The Mathematical Performance of a Selected Sample of Third Year Students in Jamaican Post-Primary Schools. Caribbean Journal of Education, Vol. 2, p.5.

(13) LEON, C.J., 1978: A Comparison of two Sequences used in Teaching Transformation Geometry by Guided Discovery. Cert. Ed. study, U.W.I. School of Education, Mona.

(14) LEO-RHYNIE, E., 1978: The Performance of Jamaican Sixth Form Students in the Cambridge 'A' Level Examinations. Caribbean Journal of Education, Vol. 5, p. 153.

(15) MACPHERSON-RUSSELL, P., CARGILL, P., & EWEKA, G., 1976: Mathematics Curriculum for Primary Schools Grades I-6. Ministry of Education, Kingston, Jamaica.

(16) MITCHELMORE, M.C., 1973: Spatial Ability and Three-Dimensional Drawing. Magazine of the Mathematical Association of Jamaica, Vol. 5, No. 2, p.15.

(17) _____ 1977 : Experiments in Individualised Instruction at a Jamaican Teachers' College. Caribbean Journal of Education, Vol.4, p. 1.

(18) _____ 1977 : Geometrical Knowledge and Intuition in Prospective Primary School Teachers. Unpublished Paper.

(19) _____ 1980 : Three-Dimensional Geometrical Drawing in Three Cultures. Educational Studies in Mathematics, Vol. 11, p.205.

(20) NOBLE, P.B., 1979: A Guided Discovery Approach to the Introduction of Angles. Cert. Ed. study, U.W.I. School of Education, Mona.

(21) O'CONNOR, D., 1975 : An Investigation of Verbal Problem-Solving in Geometry with two Selected Groups of Grade 7 Secondary School Pupils. B. Ed. study, U.W.I. School of Education, Mona.

(22) OLSON, A.T., 1975 : Mathematics Through Paper Folding. National Council of Teachers in Mathematics, Reston, Va.

(23) REID-WAUGH, P.A., 1977: An Activity Approach to the teaching of Geometry in the First Form. Dip. Ed. study, U.W.I. School of Education, Mona.

(24) REYS, R.E. & POST, T.R., 1973: The Mathematics Laboratory: Theory to Practice. Prindle, Weber & Schmidt, Boston, Mass.

(25) SCHAAF, W.L., 1976 : The High School Mathematics Library. National Council of Teachers of Mathematics, Reston, Va.

(26) SCHOOLS COUNCIL, 1966: Mathematics in the Primary Schools. HMSO, London.

(27) SHIPPEY, C.A., 1979: A Guided Discovery Approach to the Teaching of Transformation Geometry. Cert. Ed. study. U.W.I. School of Education, Mona.

(28) WALTER, M.I., 1970: Boxes, Squares and Other Things. National Council of Teachers of Mathematics.

(29) WIRZUP, I., 1976: Breakthroughs in the Psychology of Learning and Teaching Geometry. In J.L. Martin (Ed.), Space and Geometry. ERIC Centre for Science, Mathematics and Environmental Education, Columbus, Ohio.

APPENDIX B

LITERATURE MADE AVAILABLE FOR CONSULTATION

Books for pupils

Bell, S.E. Curves; Pattern, area and perimeter; Rotation and angles; Scale drawing and surveying. London: Longmans, Green, 1968.

Broomes, D.R. (Ed.) Measurement of area; Measurement of length. Port of Spain, Trinidad: Longman Caribbean, 1975.

Bruni, J.V. Experiencing geometry. California: Wadsworth, 1977.

Fielker, D.S. Cubes. Cambridge: Cambridge University Press, 1969.

Fletcher, D & Ibbotson, J. Geometry one, Geometry two, Geometry three. Edinburgh: Holmes - McDougall, 1965.

Gawronski, J.D., Hansen, V.P., Hendrickson, A.D., Jackson, R.L. & Johnson, D.A. Laboratory mathematics. Booklets 1-7. Glenview, Illinois: Scott, Foresman, 1975.

Harries, C. The geometry of the point and line. London: Longmans, Green, 1965.

Holiday, E. Altair design. Book 1. London: Longman, 1977.

Lackie, L. Understanding shapes & solids. London: Nelson, 1967.

Leedham, J., & Parker, D.V. Area and volume. London: Longmans, Green, 1966.

Mold, J. Solid models; Rolling; Triangles. Cambridge: Cambridge University Press, 1973.

Paling, D., Banwell C.S., & Saunders K.D. Creative design; Movement and pattern; Making models; Practical design. Glasgow: Oxford University Press 1971.

Pearcy, J.F.F., & Lewis, K. Experiments in mathematics. Stages 1, 2 & 3. London: Longmans, Green, 1967.

Tremblay, C.W. (Ed.) Motion Geometry (UICSM). Books 1-4 and Activities Handbook. New York: Harper & Row, 1969.

Waterfield, L.N. Introducing Geometry. London: Collins, 1968.

Travis, J. Directions, bearings and inclines. Bristol: Arnold, 1970.

Travis, J. Angles and polygons. Bristol: Arnold, 1970.

Geometry for teachers

Bell, A. & Fletcher, T. Symmetry groups. Nelson, Lancashire: Association of Teachers of Mathematics, 1964.

Coxeter, H.S.M. Introduction to geometry. New York: Wiley, 1969.

Hartung, M.L. & Walch, R. Geometry for elementary teachers. Glenview, Illinois: Scott, Foresman, 1970.

Rosen, I. Symmetry discovered. Cambridge: Cambridge University Press, 1975.

Geometry curriculum and methodology

Beckenbach, E.F. (Ed.) Informal geometry. Washington, D.C.: National Council of Teachers of Mathematics, 1969.

Brydegaard, M., & Inskeep, J.E. (Eds.) Readings in geometry from the Arithmetic Teacher. Washington, D.C.: The National Council of Teachers of Mathematics, 1971.

Gillon, E.V. (Ed.) Geometry design and ornament. New York: Dover, 1960.

Henderson, K.B. (Ed.) Geometry in the mathematics curriculum. Reston, Virginia: The National Council of Teachers of Mathematics, 1973.

Mathematics for the Majority Project. Geometry for enjoyment. Exeter: Schools Council, 1972.

Nuffield Mathematics Project. Environmental geometry. Glasgow: Chambers & London: John Murray, 1969.

Olson, A.T. Mathematics through paper folding. Reston, Virginia: The National Council of Teachers of Mathematics, 1975.

Ontario Institute for Studies in Education. Geometry: kindergarten to grade thirteen. Toronto: The Institute, 1967.

Ranucci, E.R. & Teeters, J.L. Creating Escher-type drawings. Palo Alto, California: Creative Publications, 1977.

Rice, S. Getting started in prints and patterns. New York: Bruce, 1971.

Walter, M.I. Boxes, squares and other things. Reston, Virginia: The National Council of Teachers of Mathematics, 1970.

Wenninger, M.J. Polyhedron models for the classroom. Reston, Virginia: National Council of Teachers of Mathematics, 1975.

Willson, W.W. The mathematics curriculum: Geometry. Glasgow: Blackie, 1977.

Mathematics curriculum and methodology

Biggs E.E. & MacLean, J.R. Freedom to learn. Don Mills, Ontario: Addison - Wesley, 1969.

Cohen, D. Inquiry in mathematics via the geo-board. New York : Walker, 1967.

Kidd K.P., Myers S.S. & Cilley D.M. The Laboratory approach to mathematics. Chicago : Science Research Associates, 1970.

MacPherson-Russell, P., Cargill, P. & Eweka, G. Mathematics curriculum for primary schools Grades 1-6. Kingston, Jamaica : Ministry of Education, 1976.

Mathematical Association. Primary mathematics : a further report. London : The Association, 1970.

McKillip, W.D., Cooney, T.J., Davis, E.J. & Wilson, J.W. Mathematics instruction in the elementary grades. Morristown, N.J.: Silver Burdett, 1978.

McKillip, W.D. & Davis, E.J. Mathematics instruction: Early childhood. Morristown, N.J. : Silver Burdett, 1980.

Nuffield Mathematics Project. Shape and size. Books 1, 2, 3. Glasgow: Chambers & London: John Murray, 1968.

Nuffield Foundation/CEDO. Mathematics - the first three years. Glasgow: Chambers & London: John Murray, 1970.

Nuffield Foundation/CEDO. Mathematics - the later primary years. Glasgow: Chambers & London : John Murray, 1972.

Payne, J.N. (Ed.) Mathematics learning in early childhood. Reston, Virginia : The National Council of Teachers of Mathematics, 1975.

The Schools Council. Mathematics in primary schools. London : Her Majesty's Stationery Office, 1965.

Mathematics for the Majority Project. Mathematics from outdoors. London : Chatto & Windus, 1972.

Mathematics for the Majority Project. Crossing subject boundaries. Exeter : Schools Council, 1972.

Research reports and summaries

Kilpatrick, J. & Wirszup, I. (Ed.) Soviet studies in the psychology of learning and teaching mathematics. Volume V. The Development of spatial abilities. Chicago : University of Chicago Press, 1971.

Krutetskii, V.A. The Psychology of mathematical abilities in school children. Chicago : University of Chicago Press, 1976.

Lesh, R. (Ed.) Recent research concerning the development of spatial and geometric concepts. Columbus, Ohio : ERIC Clearinghouse for Science, Mathematics and Environmental Education, 1978.

Lovell, K. The growth of understanding in mathematics : kindergarten through grade three. New York : Holt, Rinehart & Winston, 1971.

Martin, J.L. (Ed.) Space and geometry. Columbus, Ohio : ERIC Center for Science, Mathematics and Environmental Education, 1976.

Mitchelmore, M.C. Experiments in individualised instruction at a Jamaican teachers' college. Caribbean Journal of Education, 1977, 4, 1-20. (Reprint).

Mitchelmore, M.C. Geometrical knowledge and intuition in prospective primary school teachers. Unpublished paper, U.W.I. School of Education, Mona, 1977.

Mitchelmore, M.C. Three-dimensional geometrical drawing in three cultures. Educational Studies in Mathematics, 1980, 11, 205-216. (Reprint).

Rosskopf, M.F. (Ed.) Children's mathematical concepts. New York : Teachers College Press, 1975.

Rosskopf, M.F., Steffe, L.P. & Taback, S. (Eds.) Piagetian cognitive-development research and mathematical education. Washington, D.C.: National Council of Teachers of Mathematics, 1971.

Suydam M.N. & Weaver, J.F. Using research : A key to elementary school mathematics. Columbus, Ohio : ERIC Clearinghouse for Science, Mathematics and Environmental Education, 1975.

UNESCO-UNICEF. The development of science and mathematics concepts in young children in African countries. Nairobi: UNESCO-UNICEF, 1974.